

NEW
PRINCIPLES
OF

Linear Perspective:

OR THE
ART of *DESIGNING*

ON A
PLANE

THE
REPRESENTATIONS

Of all sorts of

OBJECTS,

In a more General and Simple METHOD
than has been done before.

BY

Brook Taylor, LL.D. and R.S.S.

L O N D O N :

Printed for R. KNAPLOCK at the Bishop's Head in
St. Paul's Church-yard. MDCCXIX.

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P R E F A C E.

Considering how few, and how simple the Principles are, upon which the whole Art of PERSPECTIVE depends, and without how useful, nay how absolutely necessary this Art is to all sorts of Designing; I have often wonder'd, that it has still been left in so low a degree of Perfection, as it is found to be, in the Books that have been hitherto wrote upon it. Some of those Books indeed are very voluminous: but then they are made so, only by long and tedious Discourses, explaining of common things; or by a great number of Examples, which indeed do make some of these Books valuable, by the great Variety of curious Cuts that are in

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them;

them; but do not at all instruct the Reader, by any Improvements made in the Art it self. For it seems that those, who have hitherto treated of this Subject, have been more conversant in the Practice of Designing, than in the Principles of Geometry; and therefore when, in their Practice, the Occasions that have offer'd, have put them upon inventing particular Expedients, they have thought them to be worth communicating to the Public, as Improvements in this Art; but they have not been able to produce any real Improvements in it, for want of a sufficient Fund of Geometry, that might have enabled them to render the Principles of it more universal, and more convenient for Practice. In this Book I have endeavour'd to do this; and have done my utmost to render the Principles of the Art as general, and as universal as may be, and to devise such Constructions, as might be the most simple and useful in Practice.

In order to this, I found it absolutely necessary to consider this Subject entirely anew, as if it had never been treated of before; the Principles of the old Perspective being so narrow

narrow, and so confined, that they could be of no use in my Design: And I was forced to invent new Terms of Art, those already in use being so peculiarly adapted to the imperfect Notions that have hitherto been had of this Art, that I could make no use of them in explaining those general Principles I intended to establish. The Term of Horizontal Line, for instance, is apt to confine the Notions of a Learner to the Plane of the Horizon, and to make him imagine, that that Plane enjoys some particular Privileges, which make the Figures in it more easy and more convenient to be described, by the means of that Horizontal Line, than the Figures in any other Plane; as if all other Planes might not as conveniently be handled, by finding other Lines of the same nature belonging to them. But in this Book I make no difference between the Plane of the Horizon, and any other Plane whatsoever; for since Planes, as Planes, are alike in Geometry, it is most proper to consider them as so, and to explain their Properties in general, leaving the Artist himself to apply them in particular Cases, as Occasion requires.

My

My Design in this Book, is not to trouble the Reader with a Multitude of Examples, nor to descend to a perfect Explanation of any particular Cases; but to explain the general Principles of Perspective: Which if I have been so happy as to have done, in such a manner as may be intelligible to the Reader; I don't doubt but he will easily be inclined to pardon my Shortness: For he will find much more pleasure in observing how extensive these Principles are, by applying them to particular Cases which he himself shall devise, while he exercises himself in this Art, than he would do in reading the tedious Explanations of Examples devised by another.

I find that many People object to the first Edition that I gave of these Principles, in the little Book entituled, Linear Perspective, &c. because they see no Examples in it, no curious Descriptions of Figures, which other Books of Perspective are commonly so full of; and seeing nothing in it but simple Geometrical Schemes, they apprehend it to be dry and unentertaining, and so are loth to give themselves the trouble to read it. To
satisfy

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satisfy these nice Persons in some measure, I have made the Schemes in this Book something more ornamental, that they may have some visible Proofs of the vast Advantages these Principles have over the common Rules of Perspective, by seeing what simple Constructions, and how few Lines are necessary to describe several Subjects, which in the common Method would require an infinite Labour, and a vast Confusion of Lines. It would have been easy to have multiplied Examples, and to have enlarged upon several things that I have only given Hints of, which may easily be pursued by those who have made themselves Masters of these Principles. Perhaps some People would have been better pleased with my Book, if I had done this: but I must take the freedom to tell them, that tho' it might have amused their Fancy something more by this means, it would not have been more instructive to them: for the true and best way of learning any Art, is not to see a great many Examples done by another Person; but to possess ones self first of the Principles of it, and then to make them familiar, by exercising ones self in the Practice

Practice. For it is *Practice* alone, that makes a Man perfect in any thing.

The Reader, who understands nothing of the Elements of Geometry, can hardly hope to be much the better for this Book, if he reads it without the Assistance of a Master; but I have endeavour'd to make every thing so plain, that a very little Skill in Geometry may be sufficient to enable one to read this Book by himself. And upon this occasion I would advise all my Readers, who desire to make themselves Masters of this Subject, not to be contented with the Schemes they find here; but upon every Occasion to draw new ones of their own, in all the Variety of Circumstances they can think of. This will take up a little more Time at first; but in a little while they will find the vast Benefit of it, by the extensive Notions it will give them of the Nature of these Principles.

The Art of Perspective is necessary to all Arts, where there is any occasion for Designing; as Architecture, Fortification, Carving, and generally all the Mechanical Arts; but it is more particularly necessary to the Art of Painting, which can do nothing
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without it. A Figure in a Picture, which is not drawn according to the Rules of Perspective, does not represent what is intended, but something else. So that it seems to me, that a Picture which is faulty in this particular, is as blameable, or more so, than any Composition in Writing, which is faulty in point of Orthography, or Grammar. It is generally thought very ridiculous to pretend to write an Heroic Poem, or a fine Discourse upon any Subject, without understanding the Propriety of the Language wrote in; and to me it seems no less ridiculous for one to pretend to make a good Picture without understanding Perspective: Yet how many Pictures are there to be seen, that are highly valuable in other respects, and yet are entirely faulty in this point? Indeed this Fault is so very general, that I cannot remember that I ever have seen a Picture, that has been entirely without it; and what is the more to be lamented, the Greatest Masters have been the most guilty of it. Those Examples make it to be the less regarded; but the Fault is not the less, but the more to be lamented, and deserves the more

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X P R E F A C E.

Care in preventing it for the future. The great Occasion of this Fault, is certainly the wrong Method that generally is used in the Education of Persons to this Art: For the Young People are generally put immediately to Drawing, and when they have acquired a Facility in that, they are put to Colouring. And these things they learn by rote, and by Practice only; but are not at all instructed in any Rules of Art. By which means when they come to make any Designs of their own, tho' they are very expert at drawing out, and colouring every thing that offers it self to their Fancy; yet for want of being instructed in the strict Rules of Art, they don't know how to govern their Inventions with Judgment, and become guilty of so many gross Mistakes, which prevent themselves, as well as others, from finding that Satisfaction, they otherwise would do, in their Performances. To correct this for the future, I would recommend it to the Masters of the Art of Painting, to consider if it would not be necessary to establish a better Method for the Education of their Scholars, and to begin their Instructions with the Technical Parts

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Parts of Painting, before they let them loose to follow the Inventions of their own uncultivated Imaginations.

The Art of Painting, taken in its full Extent, consists of two Parts; the Inventive, and the Executive. The Inventive Part is common with Poetry, and belongs more properly and immediately to the Original Design (which it invents and disposes in the most proper and agreeable manner) than to the Picture, which is only a Copy of that Design already formed in the Imagination of the Artist. The Perfection of this Art of Painting depends upon the thorough Knowledge the Artist has of all the Parts of his Subject; and the Beauty of it consists in the happy Choice and Disposition that he makes of it: And it is in this that the Genius of the Artist discovers and shews it self, while he indulges and humours his Fancy, which here is not confined. But the other, the Executive Part of Painting, is wholly confined, and strictly tied to the Rules of Art, which cannot be dispensed with upon any account; and therefore in this the Artist ought to govern himself intirely by the Rules of Art,

not to take any liberties whatsoever. For any thing that is not truly drawn according to the Rules of Perspective, or not truly colour'd, or truly shaded, does not appear to be what the Artist intended, but something else. Wherefore if at any time the Artist happens to imagine, that his Picture would look the better, if he should swerve a little from these Rules, he may assure himself, that the Fault belongs to his Original Design, and not to the Strictness of the Rules; for what is perfectly agreeable and just in the real Original Objects themselves, can never appear defective in a Picture, where those Objects are exactly copied.

Therefore to offer a short Hint of the Thoughts I have sometime had upon the Method which ought to be follow'd in instructing a Scholar in the Executive Part of Painting; I would first have him learn the most common Effections of Practical Geometry, and the first Elements of Plain Geometry, and common Arithmetic. When he is sufficiently perfect in these, I would have him learn Perspective. And when he has made some progress in this, so as to have prepared his
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Judgment with the right Notions of the Alterations that Figures must undergo, when they come to be drawn on a Flat, he may then be put to Drawing by View, and be exercised in this along with Perspective, till he comes to be sufficiently perfect in both. Nothing ought to be more familiar to a Painter than Perspective; for it is the only thing that can make the Judgment correct, and will help the Fancy to invent with ten times the ease that it could do without it. For the Colouring; before the Young Artist is employ'd in copying of Pictures, where there are great Variety of Colours to be imitated, it would be well that he should be instructed in the Theory of the Colours; that he should learn to know their particular Properties, their different Relations, and the various Effects that are produced by their Mixture; and that he should be made well acquainted with the Nature of the several material Colours that are used in Painting. These things ought to be learnt in a regular Method; and the Artist ought not to depend intirely upon the several indigested Observations, that may occur to him in Practice. And to this

last

last Purpose of Colouring, I cannot help thinking, that the Theory I have endeavour'd to explain in the Appendix, from Sir Isaac Newton, may be of very great use to Learners. There may be regular Methods also invented for teaching the Doctrine of Light and Shadow; and other Particulars relating to the Practical Part of Painting, may be improved and digested into proper Methods for instructing the Young Artists. But I only hint at these things, recommending them to the Masters of the Art to reflect and improve upon.

The Book it self is so short, that I need not detain the Reader any longer in the Preface, by giving him a more particular Account of what he may expect to find in it.





LINEAR PERSPECTIVE.

PART I. DEFINITIONS.

DEFIN. I.

LINEAR PERSPECTIVE is the Art of describing exactly, on a Plane Surface, the Representations of any given Objects.



TO have a compleat and clear Notion of the Principles of this Art, let the Reader consider, that a Picture drawn in the utmost degree of Perfection, and placed in a proper Position, ought so to appear to the Spectator, that he should not be able to distinguish what is there represented, from the real original Objects actually placed

placed where they are represented to be. In order to produce this Effect, it is necessary that the Rays of Light ought to come from the several Parts of the Picture to the Spectator's Eye, with all the same Circumstances of Direction, Strength of Light and Shadow, and Colour, as they would do from the corresponding Parts of the real Objects seen in their proper Places. Thus (*Fig. 1.*) supposing the Spectator, whose Eye is O, to be looking on the Picture of a Cube *abcde*, and *ABCDE* to be the real original Cube, actually placed where it seems to him to be; the Light from any Point *a* of the Picture ought to come to the Spectator's Eye O by the Ray *ao*, in the same Direction, with the same Colour, and with the same Strength of Light and Shadow, as it would do from the corresponding Point A of the Original Cube, by the Ray *AO*. The three Circumstances just mention'd make the executive Part of the Art of Painting to consist of three Parts, *viz.* Drawing, (which relates wholly to the Position, and consequently to the Shapes of the Figures on the Picture; and which when it is done exactly by Mathematical Rules, and not by an acquired Habit of the Hand and Eye, is *Perspective*, the Subject of the present Discourse;) Colouring, and the Art of Light and Shadow, which the *Italians* call the *Chiaroscuro*. The Principles of all these Parts of Painting are to be drawn from this general Consideration, and particularly those of *Perspective*. Wherefore in the Demonstrations of the following Propositions in this Book, we must always have recourse to this general Foundation, by shewing that the Rays of Light will come

come in the same Directions from the several Parts assigned in the Picture, as they would do if they came from the corresponding Parts of the original Objects placed in their proper Situations.

DEFIN. II.

When Lines drawn according to a certain Law from the several Parts of any Figure, cut a Plane, and by that Cutting or Intersection describe a Figure on that Plane, that Figure so described is called the *Projection* of the other Figure. The Lines producing that Projection, taken all together, are called the *System of Rays*. And when those Rays all pass thro' one and the same Point, they are called the *Cone of Rays*. And when that Point is consider'd as the Eye of a Spectator, that System of Rays is called the *Optic Cone*.

DEFIN. III.

When the System of Rays are all parallel to each other, and perpendicular to the Horizon, and the Projection is made on a Plane parallel to the Horizon, it is called the *Ichthyography* of the Figure proposed.

posed. Thus (*Fig. 2.*) the Plane *GHIK* being parallel to the Horizon, and the Rays *Aa*, *Bb*, *Cc*, &c. coming from the several Parts of the *Octaedron* *ABCDEF*, being perpendicular to it, and parallel to each other, the Projection *abcde* made by them is the *Ichnography* of the Figure *ABCDEF*.

DEFIN. IV.

When the System of Rays are parallel to each other, and to the Horizon, and the Projection is made on a Plane perpendicular to those Rays, and to the Horizon, it is called the *Orthography* of the Figure proposed. Thus (*Fig. 2.*) the Rays *Aa*, *Bb*, *Cc*, &c. being parallel to the Horizon, and to each other, and the Plane *GHLM*, on which the Projection is made, being perpendicular to them, *abcdef* is the *Orthography* of the Figure *ABCDEF*.

These are the common Definitions of the Terms *Ichnography* and *Orthography*, but we shall hereafter use them to signify any two Projections that are made by Systems of Parallel Rays, when those Systems are perpendicular to each

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each other, and to the Planes on which the Projections are made, as in the present Figure, without having any regard to their Situation with respect to the Horizon.

In this kind of Projections, the Projection of any particular Point, or Line, is sometimes called the Seat of that Point, or Line, on the Plane of the Projection. Thus *a* is the Seat of the Point A on the Plane GHIK, and *af* is the Seat of the Line AF on the Plane GHLM.

DEFIN. V.

When the Projection is made by a Cone of Rays, it is called the *Schenography*. Thus (*Fig. 1.*) the Figure *abcde* projected on the Plane FGH I by the Rays AO, BO, CO, &c. coming from the several Parts of the Cube ABCDE to the Point O, is the *Schenography* of the Figure ABCDE.

We shall hereafter shew this Projection to be the Picture of the Object ABCDE, to be seen by a Spectator's Eye at O.

It is also evident that the Shadows of Figures are this Projection, when the Light is consider'd as a single Point. Tho' in the Case of the Sun or Moon, that Point being at an infinite Distance (as to all Sense) the Projecting Rays are parallel to each other.

DEFIN. VI.

The *Point of Sight* is that Point, where the Spectator's Eye ought to be placed, to look upon the Picture.

This Point is no other than the Vertex of the Optic Cone, as will be evident from *Theor. 2.* where we shew, that the Representation of any Object is no other than its *Schenographic* Projection on the Plane of the Picture.

DEFIN. VII.

If from the Point of Sight there be drawn a Line perpendicular to the Picture, the Point where that Line cuts the Picture is called the *Center* of the Picture. And the Distance between that Center and the Point of Sight, is called the *Distance* of the Picture.

DEFIN. VIII.

If thro' the Point of Sight there be imagined to pass a Plane parallel to the Picture, that Plane is called the *Directing Plane*.

DEFIN.

DEFIN. IX.

By *Original Object* (whether it be Point, Line, Surface or Solid) we mean the real Object placed in the Situation it is represented to have by the Picture.

DEFIN. X.

By *Original Plane*, we mean the Plane wherein is situated any Original Point, Line, or Plane Figure.

DEFIN. XI.

The Point where any Original Line (continued if need be) cuts the Picture, is called simply the *Intersection* of that Line.

DEFIN. XII.

The Line wherein any Original Plane cuts the Picture, is called simply the *Intersection* of that Original Plane.

DEFIN. XIII.

The Point where any Original Line cuts the Directing Plane is called the *Directing*

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ing Point of that Original Line. And a Line drawn thro' that Directing Point, and the Point of Sight, is call'd the *Director* of that Original Line.

DEFIN. XIV.

The Line wherein any Original Plane cuts the Directing Plane, is call'd the *Directing Line* of that Original Plane.

DEFIN. XV.

A Line drawn thro' the Point of Sight parallel to any Original Line, is called simply the *Parallel* of that Original Line.

DEFIN. XVI.

A Plane passing thro' the Point of Sight parallel to any Original Plane, is called simply the *Parallel* of that Original Plane.

DEFIN. XVII.

The Point where the Parallel of any Original Line cuts the Picture, is call'd the *Vanishing Point* of that Line. And the Distance

Distance between that Vanishing Point and the Point of Sight, is called simply the *Distance* of that Vanishing Point.

DEFIN. XVIII.

The Line wherein the Parallel of any Original Plane cuts the Picture, is called the *Vanishing Line* of that Plane. And if from the Point of Sight there be drawn a Line cutting that Vanishing Line at Right Angles, the Point where that Vanishing Line is so cut, is called the Center of it. And the Distance between that Center and the Point of Sight, is called simply the *Distance* of that Vanishing Line.

DEFIN. XIX.

The Representation of any Figure is called the *Projection* of that Figure.

In order to comprehend the Sense of these Definitions more fully, let the Reader imagine the Plane ABC (*Fig. 3.*) to be the Surface of the Picture, O to be the Point of Sight (*Def. 6.*) the Plane ODE to be the Directing Plane parallel to the Picture (*Def. 8.*) FG to be an Original Line (*Def. 9.*) in the Original Plane FGH (*Def. 10.*) cutting

cutting the Picture in BI, and the Directing Plane in DE. And let OAC be a Plane parallel to the Original Plane FGH, and cutting the Picture in the Line AC; and let the Line OV be parallel to the Original Line FG, and cut the Picture in V. And let FG cut the Picture in B, and the Directing Plane in D. These things being supposed, B is the Intersection of the Original Line FG (*Def.* 11.) D its Directing Point (*Def.* 13.) and V its Vanishing Point (*Def.* 17.) and OD its Director (*Def.* 13.) and OV is the Distance of the Vanishing Point V (*Def.* 17.) BI is the Intersection (*Def.* 12.) DE the Directing Line (*Def.* 14.) OAC the Parallel (*Def.* 16.) and AC the Vanishing Line (*Def.* 18.) of the Original Plane FGH. And if there be drawn OS cutting the Vanishing Line AC at Right Angles in S, that Point S will be the Center (*ib.*) and SO will be the Distance (*ib.*) of the Vanishing Line AC.

AXIOM I.

The common Intersection of two Planes is a straight Line.

AXIOM II.

If two straight Lines meet in a Point, or are parallel to one another, there may be a Plane passing thro' them both.

AXIOM III.

If three straight Lines cut one another, or if two of them being parallel, are both cut by the third,

third, they will all three be in the same Plane; that is, a Plane passing thro' any two of them will also pass thro' the third.

AXIOM IV.

Every Point in any straight Line, is in any Plane that Line is in.

LEMMA I.

If BSO (Fig. 4.) and AEBD be two Planes, cutting each other in the Line ASB, and from any Point O of one of the Planes be drawn two Lines OS and OC cutting the Line AB, and the other Plane AEBD at Right Angles in S and C, and there be drawn CS; that Line CS will be perpendicular to ASB.

This follows from *Prop. II. Lib. II. Elem.*

THEOREM I.

A Line drawn from the Center of the Picture to the Center of a Vanishing Line, is perpendicular to that Vanishing Line.

DEMONSTRATION.

Imagine AEBD (Fig. 4.) to be the Plane of the Picture, O to be the Point of Sight, and OSB to be a Parallel Plane producing the Vanishing Line ASB by its Intersection with the Picture; and let S be the Center of that Vanishing Line, and C be the Center of the Picture. Then having

D
drawn

drawn OS and OC, draw CS. OS is perpendicular to AS (by *Def.* 18.) and OC is perpendicular to the Plane AEBD (by *Def.* 7.) Therefore CS is perpendicular to AS (by *Lem.* 1.) Which was to be proved.

COROL. The Distance OS of any Vanishing Line ASB, is the Hypothenufe of a Right-angled Triangle, whose Legs are the Distance of the Picture OC, and the Distance CS between the Center of that Vanishing Line and the Center of the Picture.

THEOR. II.

The Perspective Representation, or Projection, of any Object, is the same as the Ichnographic Projection of it on the Plane of the Picture, the Point of Sight being the Vertex of the Optic Cone.

DEMONSTRATION.

For by the Explanation of the Principles of the Art of Painting, in *Def.* 1. since the Light must come to the Spectator's Eye O (*Fig.* 1.) in the same Direction from any Point *a* of the Projection, as it would do from the corresponding Point A of the original Object, it is evident that the Rays *aO* and *AO* are in one and the same straight Line. Whence it is evident, that the Projection *a* is the Intersection of the Picture with the Ray *AO*, and the whole Projection *abcde* is the Schenographic Projection of the original Figure ABCDE made by the Optic Cone OABCDE, whose

whose Vertex is the Point of Sight O. Which was to be proved.

COROL. 1. The Projection of a straight Line is a straight Line. For the Optic Cone OAB, which produces the Projection *de* of any Line DE, is the Surface of a Plane Triangle ODE, all the Rays going to O from the several Points of the Line DE being in the Plane passing thro' the Lines DO and EO. Therefore *de* is the Intersection of the Picture with the Plane Triangle ODE, and consequently is a straight Line (by *Ax. 1.*)

COROL. 2. The Original of a Projection may be any Object, that will produce the same Cone of Rays. Thus the Original of the Projection *de*, may be any Line *de*, which produces the Optic Cone ODE, as well as the Line DE.

This being so, it may reasonably be ask'd, whence it comes that Figures drawn on a Picture appear to be what they are designed to represent. The Reason is, because the Mind has got a Habit of judging Objects, that are so and so related, have such and such Colours, and are so and so enlightned and shaded, to be of such and such a Shape, and to be so and so situated. These Circumstances are all of them necessary to make a Picture compleat, tho' the simple Drawing is sometimes almost sufficient, upon account of the Relation of the Parts; as in a Pavement, where all the Stones appear to be square, tho' they are represented by very irregular Figures. I say, it is the Relation of the Parts which produces this Effect; for the Representation of any one of the

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single Squares would hardly appear to be square, if there were no other Objects to bias the Judgment by their Relation to it.

THEOR. III.

The Projection of a straight Line not parallel to the Picture, passes thro' both its Intersection and Vanishing Point.

DEMONSTRATION.

All other things in *Fig. 3.* being understood (as above at the End of the Definitions) let fg be the Projection of the Original Line FG , FO , and GO being the Rays which produce the Projections f and g of the Points F and G (See *Th. 2.*) By *Theor. 2.* fg is the Intersection of the Picture with the Plane of the Triangle OFG . But all the Line FGB is in that Plane; and consequently the Intersection B . Wherefore fg continued must pass thro' B . Which was first to be proved.

OV being parallel to FG (*Def. 17.*) is in the same Plane of the Triangle OFG . Therefore fg continued will pass thro' V . Which was also to be proved.

This Theorem being the principal Foundation of all the Practice of *Perspective*, the Reader would do well to make it very familiar to him. To help him a little in his Reflections upon it, I have again represented the Sense of it in *Fig. 1.* where the Projection bc meets the Original Line BC in its Intersection K , and passes also thro' its Vanishing Point V , which is produced by its Parallel OV .
N. B.

N.B. When the Original Line it self passes through its Vanishing Point, the whole Projection of it will be that Point; so that in that case the Line may be said to vanish. This is one Reason for my using that Term. Another Reason is, that the further any Object is off, upon any Line, the smaller is its Projection, and at the same time, the nearer to this Point; and when it comes into this Point, its Magnitude vanishes, because the Original Object is at an infinite Distance. This is easily conceived by imagining a Man to be going from you in a long Walk, who appears to be smaller and smaller, the further he goes. The Reason of this Diminution will appear from the following *Corollaries*.

COROL. 1. The Projections of all Original Lines that are parallel to one another, but not to the Picture, pass thro' the same Vanishing Point: For they have but one Parallel common to them all, and consequently but one Vanishing Point. This may be seen represented in *Fig. 1.* where the Projection *da* and *cb* of the Parallel Lines *DA* and *CB* meet in their common Vanishing Point *V*.

COROL. 2. The Center of the Picture is the Vanishing Point of Lines perpendicular to the Picture. (See *Defin. 7, 15, & 17.*)

THEOR. IV.

The Projection of a Line parallel to the Picture, is parallel to its Original.

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DEMONSTRATION.

Let the Plane EF (*Fig. 5.*) be the Picture, AB be the Original Line parallel to it, and ab its Projection; O being the Point of Sight, and OAB the Optic Cone. By *Theor. 2.* ab is the Intersection of the Picture with the Plane of the Triangle OAB . Therefore AB being parallel to the Plane EF , ab is parallel to AB . For they are both in the Plane of the Triangle OAB , and do not meet; for if they did, their common Intersection would be in the Plane EF , and consequently AB would not be parallel to the Plane EF .

COROL. 1. The Projections of several Lines parallel to one another and to the Picture, are parallel to one another. Thus ab and dc are parallel to each other, and to their Originals AB and CD .

COROL. 2. The Projection $abcd$ of any Plane Figure $ABCD$ parallel to the Picture, is similar to its Original. For having drawn the Diagonal AC , and its corresponding Projection ac , the Sides ab , bc , ac , are parallel to their corresponding Originals AB , BC , AC ; wherefore the Angles at a , b , and c , are equal to the corresponding Angles at A , B , and C ; and consequently the Triangle abc is similar to the Triangle ABC . For the same reason acd is similar to ACD ; and consequently the Figure $abcd$ is similar to $ABCD$.

COROL. 3. In the same case, the Length of any Line ab in the Projection, is to the Length of

of its Original AB , as the Distance of the Picture is to the Distance between the Point of Sight and the Plane of the Original Figure. Let OgG be perpendicular to those two Planes, cutting them in g and G . Then will $ab : AB :: Oa : OA :: Og : OG$ (by *Prop. 17. Lib. 11. Elem.*) But g is the Center, Og is the Distance of the Picture, and OG is the Distance between the Point of Sight O , and the Original Plane $ABCD$. Wherefore ab is to AB , as the Distance of the Picture is to the Distance between the Point of Sight and the Plane of the Original Figure.

THEOR. V.

The Projection of a Line is parallel to its Director.

In *Fig. 3.* as already explain'd at the End of the Definitions, and in *Theor. 3.* the Lines OF , OG , OD , fg , are all in the same Plane. But the Directing Plane ODE is parallel to the Plane of the Picture $ABIC$ (*Def. 8.*) Therefore the Director OD is parallel to the Projection fg (by *Prop. 16. Lib. 11. Elem.*)

COROL. 1. The Projections of Lines that have the same Director, are parallel to each other.

COROL. 2. When the Original Line is parallel to the Picture, its Director is parallel to it, and consequently is in the Parallel of any Plane passing thro' that Original Line; and therefore the Vanishing Line of that Plane, and the Projection of the Line, are parallel to one another.

THEOR.

THEOR. VI.

The Vanishing Line, Intersection, and Directing Line of any Original Plane, are parallel to each other.

In the same *Fig.* understood as has been already explain'd, the Planes OVC and DFH being parallel (*Def.* 16.) as also ODE and CAB (*Def.* 8.) the Vanishing Line CV, the Intersection IB, and the Directing Line ED, are parallel to each other (by *Prop.* 16. *Lib.* 11. *Elem.*) Which was to be proved.

COROL. The Distance fV between the Projection of any Point, and the Vanishing Point V, is to the Distance BV, between the Intersection B, and the same Vanishing Point V, as the Distance OV of that Vanishing Point V, is to the Distance DF, between the Director and the Original Point F. For OVB D is a parallelogram, wherefore BV is equal to DO, and the Triangles fOV and OFD are similar, because their Sides Vf and OD are parallel. Wherefore $fV : VO :: DO (=BV) : DF$.

THEOR. VII.

The Vanishing Points of all Lines in any Original Plane, are in the Vanishing Line of that Plane.

DEMON-

DEMONSTRATION.

For since all the Original Lines are in the same Plane, their Parallels, which all pass thro' the Point of Sight, will be all of them in the Parallel Plane (by *Prop. 15. Lib. II. Elem.*) wherefore all the Vanishing Points are in the Vanishing Line.

COROL. 1. Original Planes, that are parallel, have the same Vanishing Line.

COROL. 2. The Vanishing Point of the common Intersection of two Original Planes, is the Intersection of their Vanishing Lines.

COROL. 3. The Vanishing Line of a Plane perpendicular to the Picture, passes through the Center of the Picture.

THEOR. VIII.

The Intersections of all Lines in the same Original Plane, are in the Intersection of that Plane.

This needs no *Demonstration*.

COROL. 1. The Intersection of the common Intersection of two Original Planes, is the Intersection of their Intersections.

COROL. 2. Planes, whose common Intersection is parallel to the Picture, have parallel Intersections, and also parallel Vanishing Lines.

PROBLEM I.

Having given the Center and Distance of the Picture, to find the Projection of a Point, whose Seat on the Picture, with its Distance from it, are given.

Let S (Fig. 6.) be the Center of the Picture, and b the given Seat of the Original Point. Draw at pleasure SO equal to the Distance of the Picture, and parallel to it draw bA equal to the Distance of the Original Point from its Seat. Draw Sb and AO meeting in a , which will be the Projection sought.

DEMONSTRATION.

If the Angle OSb (and consequently Aba) had been a Right Angle; then turning the Triangles SOa and bAa round the Line Sab as an Axis, till SO and bA become perpendicular to the Picture, O would be the Point of Sight, and A the Original Point, and AO would be the Visual Ray cutting the Picture in a , which consequently would be the Projection of the Point A , by *Theor.* 2. But the Point a is the same, whether the Angle OSb be a right Angle or not, because the Triangles OSa and Aba are similar, and therefore $Sa:ab::SO:bA$, which Proportionality is not affected by altering the Angle OSb . Therefore in all cases the Point a thus found, is the Projection sought of a Point, whose
Seat

Seat on the Picture is b , and its Distance from its Seat is bA .

COROL. 1. Having drawn Sb , the Point a may be found by a Scale and Compasses dividing the Line Sb in a , so that Sa may be to ab , as the Distance of the Picture SO , is to the Distance bA of the Original Point from its Seat b .

COROL. 2. By this Proposition the Projection of any Line may be found, by finding the Projections of two Points in it, and drawing a Line thro' those two Projections.

PROBLEM II.

To find the Projection of a Line, its Vanishing Point and Distance, having given its Seat, Intersection, and the Angle it makes with its Seat, and the Center and Distance of the Picture.

Let DE Fig. 6. be the given Seat of the Line proposed, D its Intersection, and S the Center of the Picture. Draw DC making the Angle EDC equal to the Angle the Original Line makes with its Seat. Draw SV parallel to DE , and SO perpendicular to it, and equal to the Distance of the Picture. Then draw OV parallel to DC cutting SV in V , and draw DV . Then will V be the Vanishing Point and OV its Distance, and DV the indefinite Representation of the Line proposed.

DEMONSTRATION.

Imagine the Planes OSV and CED to be turned round the Lines SV and DE as Axes, till they become perpendicular to the Picture. Then will O be the Point of Sight, and DC the Original Line; and OV being parallel to it, V is its Vanishing Point (by *Def.* 17.) and consequently DV is its Projection (by *Theor.* 3.) Which was to be proved.

COROL. 1. DAC being conceived as the Original Line laid on the Picture, by turning the Plane CDE round the Line ED, the Projection of any part of it AC may be found by drawing the Lines AO and CO as Visual Rays, cutting DV in *a* and *c*. For the Points *a* and *c* depend only on the Parallelism of the Lines OV and DC, and their proportion; aV being to aD as VO is to DA , and $cV : cD :: VO : DC$, upon account of the similar Triangles aVO and aDA , and cVO and cDC . To understand this more clearly, the Reader may compare this Figure with *Fig.* 3. where the Points O, V, *f*, *g*, B, G, F, are analogous to the Points O, V, *c*, *a*, D, A, C, respectively.

COROL. 2. Having found DV, the Projection *c* of any Point C may be found by a Scale and Compasses, as making $cV : cD :: OV : CD$.

PRO-

PROBLEM III.

Having given the Projection of a Line, and its Vanishing Point; to find the Projection of the Point that divides the Original Line in any given Proportion.

Let AB (*Fig. 7.*) be the given Projection of the Line to be divided, and V its Vanishing Point. Draw at pleasure VO and *ba* parallel to it, and thro' any Point O of the Line VO draw OA and OB cutting *ba* in *a* and *b*. Divide *ab* in *c* in the Proportion given, and draw Oc cutting AB in C. Then will C be the Projection sought, the Original of BC being to the Original of CA, as *bc* is to *ca*.

DEMONSTRATION.

OV being parallel to *ba*, *ba* may be consider'd as the Original Line and OV as its Parallel, and consequently O as the Point of Sight, and *a*O, *b*O, *c*O, as Visual Rays projecting the Points A, B, C.

COROL. The Mathematical Reader will easily find, that $CA \cdot BV : CB \cdot AV :: ca : cb$. Whence the Point C may be found by a Scale and Compasses, making $CA : CB :: AV \cdot ca : BV \cdot cb$.

PROBLEM IV.

Having given the Projection of a Line, and its Vanishing Point; from a given Point in that Projection, to cut off a Segment, that shall be the Projection of a given Part of the Original of the Projection given.

Let AB (Fig. 8.) be the Projection given, V its Vanishing Point, and C the Point from whence is to be cut off the Segment. Draw at pleasure VO and abc parallel to it, and from any Point Q in VO draw OA , OB , OC cutting ab in a, b, c . Make cd to ab , as the given Part is to the Original of AB , and draw Od cutting AB in D . Then will CD be the Segment sought.

DEMONSTRATION.

If QOV be conceived as the Vanishing Line of any Plane passing through the Original of ABV ; $abcd$ being parallel to it, may be conceived as the Projection of a Line parallel to the Picture (by *Cor. 2. Theor. 5.*) and therefore its Parts ab and cd will be in the same proportion to one another as their Originals (by *Theor. 4.*) But because of the Vanishing Point O , the Originals of Oa , Ob , Oc , Od are parallel (by *Cor. 1. Th. 3.*) Wherefore the Original of CD is to the Original of AB , as cd is to ab (by *Prop. 2. Lib. 6. Elem.*) Which was to be proved.

N.B. This Proposition might have been demonstrated as the foregoing, and the foregoing may

LINEAR PERSPECTIVE

may be consider'd as a particular Case of this, viz. when the Point C of this Proposition coincides with one of the Points A or B.

COROL. The Point D may be found by a Scale and Compasses, making $DC:DV::dc:AB$ $CV:ab::AV:BV$.

PROBLEM V.

Having given the Center and Distance of the Picture; to find the Vanishing Line (with its Center and Distance) of a Plane, whose Intersection is given, with the Angle of its Inclination to the Picture.

Let AB (Fig. 9.) be the given Intersection of the Plane, and C the Center of the Picture. Draw CO parallel to AB, and equal to the Distance of the Picture, and draw CA perpendicular to AB. Draw OS cutting AC in S, so that the Angle OSC may be equal to the Inclination of the original Plane to the Picture. Draw SD parallel to AB. Then will SD be the Vanishing Line sought, S its Center, and OS its Distance.

DEMONSTRATION.

Imagine the Triangle OSC to be raised up on the Picture, so that OC may be perpendicular to the Picture. In that case O will be the Point of Sight, and SD being parallel to AB, a Plane passing thro' the Line SD and the Point O will be the Parallel of any original Plane passing thro' AB,

AB, and inclined to the Picture in the Angle OSC. Wherefore SD is the Vanishing Line sought. And OS, in that supposition, being perpendicular to SD, S is the Center, and SO the Distance of the Vanishing Line SD. Which was to be proved.

N. B. Taking OC for Radius, CS is the Co-tangent, and OS the Co-secant of the Inclination of the Original Plane to the Picture.

PROBLEM VI.

Having given the Intersection of an Original Plane, with its Vanishing Line, its Center and Distance; to find the Projection of any Line in the Original Plane, having the Original Figures drawn out in their just Proportions.

Let DF (Fig. 10.) be the Intersection given, HG the Vanishing Line, and S its Center. Draw SO perpendicular to GH, and equal to the Distance of the Vanishing Line GH, and let the Space X be the Original Plane, seen on the Reverse, as Objects appear in a Looking Glass; the Space Y being the Parallel Plane in the same manner folded down on the Picture; and let AB be the Original Line, whose Projection is sought. Let AB cut the Intersection in D, and draw OG parallel to AB, cutting the Vanishing Line in G. Draw DG, which will be the indefinite Projection of AB. Thro' A and B draw at pleasure AC and BC meeting in C, and in the same manner find their indefinite Projections FI, and EH cutting DG in *a* and *b*. Then

AB

will

will ab be the determinate Projection of AB , a being the Projection of the Extremity A , and b the Projection of the Extremity B .

Otherwise. Suppose KL to be the Original Line given. Having found its indefinite Projection QG , as before, draw OK and OL cutting it in k and l , which will be the Projections of the Extremities K and L .

Otherwise, by the Directors.

Let DF (*Fig. 11.*) be the Intersection given. And let the Original Plane be folded down on the Picture, so as to bring the Directing Line into the Place HI , the Distance between AF and HI being equal to the Distance of the Vanishing Line given. Let also O be the Point of Sight brought into the Picture at the same time, along with the Directing Plane HOI . To find the indefinite Projection of any Original Line AB , continue it till it cuts EF in F and HI in G ; then draw OG , and Fa drawn parallel to it will be the indefinite Projection sought. Then finding in the same manner the indefinite Projection $E d$ of any other Line AD passing thro' A , by its Intersection with Fa is got the Projection a of the Extremity A . And in the same manner is got the other Extremity b . Or those Extremities might be found by drawing Lines from A and from B to O , as in the foregoing Construction.

DEMONSTRATION.

Imagine the Figures to be folded, *Fig. 10.* in DF and HG , and *Fig. 11.* in EF and HI , till the

F

the

the Original Plane, its Parallel, and the Directing Plane, and along with them the Point of Sight O, come into their proper Places. Then you will find, that D in *Fig. 10.* and F in *Fig. 11.* will be the Intersection of AB, and G in *Fig. 11.* will be its Directing Point. But OG in *Fig. 10.* is still parallel to AB, wherefore G is its Vanishing Point, and DG its indefinite Projection (by *Theor. 3.*) and Fa in *Fig. 11.* is still parallel to OG, which is the Director of AB; wherefore Fa is the indefinite Projection of AB (by *Theor. 5.*) That a found by the intersection of FI with DG in *Fig. 10.* and of Ed with Fa in *Fig. 11.* is the Projection of the intersection of the Original Lines AB and AC in *Fig. 10.* and of AB and AD in *Fig. 11.* is obvious. The other Construction by the Lines AO is the same as by the Lines AO and CO for finding the Points a and c in *Fig. 6.* as is explain'd in *Cor. 1. Prob. 2.*

N.B. 1. The Reader that is not used to Mathematical Subjects may help himself in conceiving the Manner here described of bringing the Original Plane, its Parallel, and the Directing Plane, with the Point of Sight, into the Plane of the Picture, by conceiving in *Fig. 3.* those several Planes to be brought together by enlarging the Angles OVB and ODB, till the Planes lie flat on one another, and the Original Plane is seen on the back-side.

N.B. 2. In *Fig. 11.* the Projections ad and kl are parallel, their Originals having both of them the same Director OH, according to *Cor. 1. Theor. 5.* The same may be observed in lm and cd, which have the same Director OI.

PRO-

PROBLEM VII.

Having given the same things as in the foregoing Problem, to find the Projection of any Figure in the Original Plane.

This is done by finding the Projections of the several Parts of the Figure given, by the foregoing Problem.

For example, the Projection $klmnp$ (*Fig. 10.*) of the Pentagon $KLMNP$ is found thus. Drawing OG, OH, OI, OV parallel to KL, LM, MN, KP respectively, the Points G, H, I , and V , are their Vanishing Points. And KL, LM, MN , being continued cut the Intersections in their Intersections Q, R, T . Whence drawing QG, RH, TI , are got the Projections l, m of the Points L and M , by their mutual intersections. Then drawing OK and ON , are got the Points k and n . Then drawing kV to the Vanishing Point V of KP , is got the indefinite Projection of KP . Lastly drawing OP is got the Point p .

The Projections of Curvelined Figures are to be got by finding the Projections of several of their Points, and afterwards joining them neatly by hand. Thus in *Fig. 13. n. 1.* DE being the Intersection, and VF the Vanishing Line, and O the Point of Sight, and ABC an Original Circle, placed as in the foregoing Problem, the Projection a of any Point A may be found by drawing at pleasure AD , and OV parallel to it; then drawing DV and OA meeting in the Point sought a , according to the Construction in *Problem 6.* D being

being the Intersection, and V the Vanishing Point of the Line AD. And the several Lines AD being drawn parallel to one another, the same Vanishing Point V may serve for them all.

Or as in N.2. VF being the Directing Line brought into the Picture, as in Fig. 11. and the rest remaining as before; drawing at pleasure AD cutting DE and VF in D and V, then drawing QV and Da parallel to it, the Projection *a* is got by drawing OA cutting Da in *a*. And the same Point V being used for all the Points A, all the Lines Da will be parallel to one another, and to the same Line OV.

PROBLEM VIII.

To find the Projection of any Figure in a Plane parallel to the Picture.

The Projection being similar to its Original (by Cor. 2. Theor. 4.) this is done by making an exact Copy of the Original Figure; making the Homologous Sides in the Proportion explain'd in Cor. 3. of the same Theorem.

PROBLEM IX.

Having given the Intersection of a Plane, and its Vanishing Line, with its Center and Distance; to find the Original of any Projection given on the Picture.

Every thing being disposed in Fig. 10. as in Problem 6, and 7. let it be proposed to find the Original

Original of the Figure $klmnp$. Having continued the Projections kl , lm , mn till they cut the Intersection and Vanishing Line in their Intersections Q, R, T, and their Vanishing Points G, H, I, and kp in its Vanishing Point V, draw OG, OH, OI, OV, and QK, RM, TN parallel to the three first respectively meeting in L and M, which will be the Originals of l and m . Draw Ok and On, which will cut QL and TM in the Originals K and N, of k and n . Then draw KP parallel to OV, and Op cutting it in P, which will be the Original of the Point p . Lastly drawing NP, you have the Original Figure sought KLMNP.

DEMONSTRATION.

This Construction is evident from *Problem 7*: it being the Reverse of it.

N. B. In the same manner one may go back to the Original Figure by the Directors, as in *Fig. 11*.

PROBLEM X.

The same things being given, to find only the Length of the Original of a Projection given.

Let III (*Fig. 10.*) be the Projection given; the rest of the Figure being to be understood as in the foregoing Problems. Continue III till it cuts the Vanishing Line in its Vanishing Point V. And draw VO. In the Vanishing Line take V3 equal to VO, and draw 3I and 3II cutting the Intersection in 1 and 2. Then will 12 be the Length sought of the Original of III.

DEMON-

DEMONSTRATION.

Let W be the Intersection of III. V_3 being equal to VO the Distance of the Vanishing Point V , and W_2 being parallel to V_3 , the Point 3 may be consider'd as the Point of Sight, and $W_{1.2}$ as the Original Line and 3.1 and 3.2, as Visual Rays producing the Projection I II.

COROL. The Length 1.2 may be found by a Scale and Compasses, making $1.2 : V_3$ (or to VO) :: $III \times WV : IV \times II$.

PROBLEM XI.

Having given the Vanishing Line of a Plane, its Center and Distance, and the Projection of a Line in that Plane; to find the Projection of another Line in that Plane, making a given Angle with the former.

Let O (*Fig. 10.*) be the Point of Sight placed as in the foregoing Problems, GH being the Vanishing Line, and ab the given Projection; it being required to draw $a'c$, so that the Original of the Angle bac may be equal to a given Angle. Continue ab to its Vanishing Point G . Draw GO , and OI making GOI equal to the given Angle, and cutting the Vanishing Line in I . Then draw Iac , which will be the Line sought.

DEMON-

DEMONSTRATION.

The Figure being understood as in the foregoing Problems, let AB be the Original of ab , and consequently be parallel to OG (by *Def. 15.* and *Theor. 3.*) For the same reason AC parallel to OI is the Original of ac , I being its Vanishing Point (by *Theor. 3.*) But AB and AC being parallel to OG and OI , the Angle BAC is equal to GOI , which is equal to the given Angle by the Construction. Wherefore bac representing the Angle BAC , represents that given Angle. Which was to be done.

N.B. If it had been required to make abc to represent the Angle ABC , the Angle GOH must have been made equal to the Complement of the Angle ABC to two Right Angles.

PROBLEM XII.

Having given the Vanishing Line of a Plane, its Center and Distance, and the Projection of one Side of a Triangle of a given Species in that Plane; to find the Projection of the whole Triangle.

The Projections of the Sides wanting are to be found by the foregoing Problem, the Angles of the Triangle being given. Thus having given the Projection ab (*Fig. 10.*) of the Side AB of the Triangle ABC , the Vanishing Point I of the Side ac is found by making the Angle IOG equal to the Angle CAB , and the Vanishing Point H of

of the Side bc is found by making the Angle HOG equal to the Complement of the Angle CBA to two Right Angles.

N. B. If the Vanishing Point of the Line given ab is out of reach, you may proceed thus. Taking any Line DR (parallel to the Vanishing Line HG by *Theor.* 6.) for the Intersection, by means of two Lines HbE, IaF, drawn at pleasure thro' b and a , find the Originals A and B of the Points a and b (by *Prob.* 9.) and draw AB. Then on the Side AB compleat the Original Triangle, and find the Projections of the Sides wanting by *Prob.* 7.

PROBLEM XIII.

Having given the Vanishing Line of a Plane, its Center and Distance, and the Projection of one Side of any Figure in that Plane, to find the Projection of the Whole Figure.

Resolve the whole Figure given into Triangles, by means of Diagonals, and find the Projections of those Triangles one after another (by *Prob.* 12.) beginning with those that have the Line given for one of their Sides.

The same thing may be done several Ways by the Application of the foregoing Problems, as is most convenient in every particular Case. This will be best understood by a few Examples.

Example I.

Fig. 14. In this Example IK is the Vanishing Line, S its Center, and SO its Distance, and AB parallel

parallel to IK is the given Projection of one side of a regular Hexagon. Having drawn OG parallel to IK. (by *Def. 15.*) the Original of AB being parallel to the Picture (by *Cor. 2. Theor. 5.*) the Vanishing Points H, I, K of the Sides and Diagonals BC, FE, AD; AF, BE, CD; AC, are found by *Prob. 11.* making the Angles HOG 60 degr. IOG 120 degr. KOG 30 degr. Then drawing AK and BH, is got the Point C; drawing AH and CI is got D; drawing DE parallel to IK, and AS, is got E (for S is the Vanishing Point of AE, the Original of the Angle EAB being a Right Angle, as is GOS.) Lastly drawing EH and AI is got F, which compleats the Figure sought.

Example II.

Fig. 15. In this Figure the Projection *mrptqs* of the Figure MRPTQS (which is the Ichnography of a Regular Icosaedron repoling on one of its Faces,) is found, having given the Projection *ab* of the Side AB, VX being the Vanishing Line, and O the Point of Sight reduced to the Picture, as in the foregoing *Problems*. The Original Ichnography is described by making two concentric and parallel regular Hexagons, AFBICH, and RMSQTP, whose homologous Sides are in the Proportion of the Parts of a Line cut in extream and mean proportion, (See *Def. 3. Lib. 6. Elem.*) and drawing the Lines, as is obvyous enough in the Figure.

Having continued *ab* to its Vanishing Point V, the Vanishing Points W and X of the other two Sides of the Triangle *abc* are found by

G

Prob.

Prob. II. Then drawing at pleasure sp parallel to VX , and drawing Wa and Wb cutting it in a and b , and dividing ab in k, d, e, l , in the same proportion as AB is divided in K, D, E, L , and draw kW, dW, eW, lW , are got the Projections k, d, e, l of the Points K, D, E, L , (by *Prob. 3.*) Then drawing dX, eW , and eX , are got the Points f , and g ; and drawing gV , are got the Points h and i . Drawing kX and lW , is got the Point m , and n (which is the Projection of N) by the Intersection of kX with dW already drawn. Then making do and op each equal to md , and drawing oW, pW , are got the Points o and p (by *Prob. 3.*) Then drawing pV is got the Point q , by its Intersection with lW already drawn. Drawing oW and nV is got r : And making ms equal to md , and drawing sW is got s . Lastly, drawing sX cutting roW already drawn, is got t . The rest is done by joining the Points found, as is evident enough in the Scheme.

Example III.

Fig. 16. In this Example is found the Projection of the Ichnography of a regular Dodecaedron, having the Projection of one Side given, by returning to the Original Figure, by *Prob. 9.* and then proceeding by *Prob. 7.* I shall leave the Reader to exercise himself in considering this Scheme, and only observe, that the Original Ichnography is made by describing two concentric and parallel Decagons, whose homologous Sides are as the Segments of a Line cut in extream and mean Proportion.

Example

Example IV.

Fig. 17. In this Figure DC being the Vanishing Line, and O the Point of Sight reduced to the Picture, as in the foregoing Problems, the Projection ANBMLP, of a regular Octaedron, having given the Projection of one of the Sides AB, is found as follows. Having continued AB to its Vanishing Point C, the Vanishing Point G of the Sides AK and NM is found by *Prob. II.* making the Angle COG equal to 60 degr. Then (by *Prob. 10.*) taking any Line *bl* (parallel to CD) for the Intersection, and making CD equal to CO, and GH equal to GO, and drawing DA and DB cutting *bl* in *a* and *b*, *ab* is the Length of the Original of AB, and drawing HA cutting *bl* in *a*, and making *al* equal to *ab*, and drawing *lH* cutting AG in L, is got the Projection AL (the Original of it being equal to *al*, and consequently to the Original of AB, *al* and *ab* being equal. Then dividing *ab* and *al* each into three equal Parts by the Points *e, f, i, k*, and drawing *eD, fD, iH, kH*, are got the Points E, F, I, K (by *Prob. 3.*) Then drawing FG, KH, EI, are got the Points M, N, P, which compleat the Figure.

PROBLEM XIV.

Having given the Center and Distance of the Picture, and the Vanishing Line of a Plane; to find the Vanishing Point of Lines perpendicular to that Plane.

G 2

Let

Let AB (*Fig. 18.*) be the Vanishing Line given, and C the Center of the Picture. Draw CA perpendicular to AB , and CO parallel to it, and equal to the Distance of the Picture. Draw AO and OD perpendicular to it, cutting CA in D ; which will be the Vanishing Point sought.

DEMONSTRATION.

Imagine the Triangle AOD to be turned up on the Plane of the Scheme, so that CO may be perpendicular to it, O being brought into the Point of Sight. This being done, the Plane passing through the Point O and the Line AB will be the Parallel of the Original Plane, and the Line OD will be perpendicular to it; and consequently will be the Parallel of Lines perpendicular to that Original Plane. Wherefore D is the Vanishing Point of those Perpendiculars (*by Def. 17.*)

N.B. 1. When the Vanishing Line AB passes through the Center of the Picture, that is, when the Original Plane is perpendicular to the Picture, the Point D will be infinitely distant, the Line OD being parallel to AD , and the Projections of the Lines perpendicular to the Plane proposed will all of them be perpendicular to AB , they being to meet the Line AC , which is perpendicular to it at an infinite distance; and consequently they will be parallel to one another. Which they ought to be upon another account, their Originals being all parallel to the Picture.

N.B. 2. But when the Original Plane is parallel to the Picture, the distance CA will be infinite, and consequently OA will be parallel to CA ,

CA, and OD will coincide with OC, making the Point D to fall into the Center of the Picture C, agreeably to *Cor. 2. Theor. 3.*

N.B. 3. CD is a third proportional to AC and CO; as also is AD to AC and AO.

N.B. 4. OD is the Distance of the Vanishing Point D.

PROBLEM XV.

Having given the Center and Distance of the Picture, to find the Vanishing Line, its Center and Distance, of Planes that are perpendicular to those Lines that have a certain given Vanishing Point.

Let C (*Fig. 18.*) be the Center of the Picture, and D the Vanishing Point given. Draw DC, and CO perpendicular to it, and equal to the Distance of the Picture. Then draw DO, and OA perpendicular to it, cutting DC in A. Perpendicular to DC draw AB, which will be the Vanishing Line sought, A being its Center (by *Theor. 1.*) and OA its Distance.

This Construction follows necessarily from the Construction of the foregoing Problem, and the Observations at the End of that may be applied here.

PRO-

PROBLEM XVI.

Having given the Center and Distance of the Picture, through a given Point to draw the Vanishing Line of a Plane that is perpendicular to another Plane whose Vanishing Line is given, and to find the Center and Distance of that Vanishing Line.

Let AB (*Fig. 18.*) be the Vanishing Line given, and C the Center of the Picture, and let E be the Point given. Find the Vanishing Point D of Lines perpendicular to the Original Planes of AB, (by *Prob. 14.*) Draw DE, which will be the Vanishing Line sought. Draw CF cutting DE at Right Angles in F, and F will be the Center of the Vanishing Line DE (by *Theor. 1.*) Make a Right-angled Triangle, whose Base is CF, and its Perpendicular is equal to the Distance of the Picture, and its Hypothenuse will be the Distance of the Vanishing Line DE (by *Cor. Th. 1.*)

DEMONSTRATION.

Because the Plane, whose Vanishing Line is sought, is perpendicular to the other Plane, its Vanishing Line must pass thro' the Vanishing Point D of Lines perpendicular to that other Plane, because some of those Lines are in the Plane sought. Therefore DE is the Vanishing Line sought. The rest needs no Demonstration.

COROL.

COROL. 1. If FC be continued till it cuts the Vanishing Line given in B , B will be the Vanishing Point of Lines perpendicular to the Original Plane of the Vanishing Line DE . For that Vanishing Point is in the Line FC by the Construction of *Problem 14.* and it is in the Vanishing Line given by the Demonstration of the present Problem.

COROL. 2. And therefore if the Vanishing Lines AB and DE meet in G , the Points B, D , and G will be the Vanishing Points of the three Legs of the solid Angle of a Cube, which are perpendicular to one another. And drawing DB, BG, GD , and DB will be the Vanishing Lines of the three Planes that contain that solid Angle.

COROL. 3. The Distance of the Vanishing Line DG is equal to the Line FP , the Point P being the Intersection of the Line FC with a Circle described on the Diameter DG .

PROBLEM XVII.

Having given the Center and Distance of the Picture, and the Vanishing Point of the common Intersection of two Planes that are inclined to one another in a given Angle, and the Vanishing Line of one of them; to find the Vanishing Line of the other of them.

Let C (*Fig. 18.*) be the Center of the Picture, BG the given Vanishing Line of one of the Planes, and B the Vanishing Point of their common

mon Intersection, and H the Angle of their Inclination to one another. Find the Vanishing Line GD , of Planes perpendicular to the Lines whose Vanishing Point is B (by *Prob. 15.*) Let that Vanishing Line cut the Vanishing Line given in G . In GD find the Vanishing Point E of Lines making the given Angle H with the Lines whose Vanishing Point is G (by *Prob. 11.*) that is, in BCF perpendicular GFD take FP equal to the Distance of the Vanishing Line GD (found by *Prob. 15.*) and draw PG , and PE making the Angle EPG equal to H . Draw BE , which will be the Vanishing Line sought.

DEMONSTRATION.

Imagine the Triangle GPE to be turned up on the Line GE , so that the Point P may be in the Point of Sight perpendicular over the Center of the Picture C . In that case the Planes BPG , GPD , DPB will be the Parallels of three Original Planes, whose Vanishing Lines are BG , GD , DB ; that whose Vanishing Line is DG being perpendicular to the other two (by the Construction, because it is perpendicular to their common Intersection; whose Vanishing Point is B .) Therefore the Original Planes, whose Vanishing Lines are BG and BD , are inclined to one another in the Angle EPG , that is, in the Angle H , (for the Inclination of two Planes is always measured in a Plane perpendicular to their common Intersection.) Therefore BG being the Vanishing Line given, BE is the Vanishing Line sought.

N.B.

N.B. The Center of the Vanishing Line BE is found by drawing a Line perpendicular to it from C (by *Theor.* 1.) and then its Distance is found, as was found the Distance PF in *Prob.* 16.

PROBLEM XVIII.

Having given the Center and Distance of the Picture, and the Vanishing Line of one Face of any solid Figure proposed, and the Projection of one Line in that Face; to find the Projection of the whole Figure.

By means of the Projection given find the Projection of the Ichnography of the Figure proposed on the Plane of that Face, whose Vanishing Line is given (by *Prob.* 13.) Then (by *Prob.* 16.) find the Vanishing Line of the Plane of the Orthography, and describe the Projection of the Orthography by the help of the Lines already given in the Ichnography (by *Prob.* 13.) Lastly, by the intersections of the Projections of Perpendiculars to the Ichnography and Orthography, will be found the several Points of the Projection required.

Otherwise. Having found the Projection of the Face whose Vanishing Line is given, by means of the Projection of the Line given, find the Vanishing Lines of the adjacent Faces (by *Prob.* 17.) and describe their Projections by the help of the Lines given in the Projection of the first Face; and so on, till the whole Projection sought is completed.

This is a general Description of the Method to be used in putting any Figures proposed into *Perspective*; but in the Practice of particular Cases several Expedients may be used, in the various Application of the foregoing Problems. But all this will be best understood by Examples.

Example V.

Fig. 19. In this Figure is found the Projection of a regular Dodecaedron, having given the Projection AB of one Side parallel to the Picture, by means of the Ichnography and Orthography; FG (parallel to AB) being the given Vanishing Line of the Face ABCDE, F its Center, H the Center of the Picture, and HO its Distance. To avoid Confusion of Lines, the Ichnography and Orthography are removed from the Space taken up by the Projection sought in the following manner. For the Ichnography; drawing at pleasure *ab* parallel to AB, and at a sufficient distance from it, and then drawing IA and IB cutting *ab* in *a* and *b*, the Line AB is transfer'd to *ab*; I being the Vanishing Point of Lines perpendicular to the Face ABCDE, whose Vanishing Line is FG (found by *Prob. 14.*) This being done, the whole Ichnography is described on the Line *ab*, by *Prob. 13.* as it is described in *Ex. 3.*

For the Orthography; IHF passing thro' the Center of the Picture, is taken for its Vanishing Line, as being most convenient; the Orthography in this case being the most simple, and the Projections of Lines perpendicular to it being all of them perpendicular to FI (by *Note 1. Prob. 14.*) Having drawn GA (from the Vanishing Point

Point G of the Line ae in the Ichnography) and eI cutting it in E, is got the Projection AE. Then drawing Aa and Ee both parallel to GF (and consequently representing perpendiculars to the Orthography, as is already said) and ae at pleasure passing thro' F, and cutting them in a and e, is got ae ; by the means of which the whole Projection of the Orthography is described (by *Prob. 13.*)

Having thus got the Projections of the Ichnography and Orthography, any Point K of the Projection sought is got by drawing kK parallel to FG and kI , from the corresponding Points k and k , meeting each other in K.

For understanding the Projection of a Cube, which appears in this Figure, after what has been already said, it is sufficient to inform the Reader, that two of its Vanishing Lines are FG, and FI, and the third is a Line passing thro' I parallel to FG.

As to the Shadows, which are supposed to be cast by the Sun on the Plane of the Face ABCDE of the Dodecaedron; the Shadow uv of any Line Vv is found as follows. S is the given Vanishing Point of all the Rays of Light, which being supposed to come from the Sun, are to be consider'd as parallel. Therefore IS passing thro' the Vanishing Point I of the Line Vv and the Vanishing Point S of the Rays, is the Vanishing Line of the Plane made by all the Rays passing thro' the Line Vv, and projecting the Shadow vu . And IS cutting the Vanishing Line FG of the Plane the Shadow is cast on in s, s is the Vanishing Point of the Shadow vu , which is the

Intersection of the Plane of the Shade (whose Vanishing Line is Is) and the Plane the Shadow is cast on (s being the Vanishing Point of that Intersection by *Cor. 2. Th. 7.*) Having therefore drawn vs , and VS cutting it in u , vu is the Shadow of the Line vV .

N.B. In the Original of the Orthography in the present Figure, the Points e, m, s, p are the Angles of a Square. The Lines on, al, qr being parallel to em , and oq, nr being parallel to ep , and equal to al . tk, on, qr , are equal, and on, em, al , are in the continued Geometrical Proportion, of the lesser Segment to the greater, of a Line cut in extream and mean Proportion.

Several Lines mention'd in this Scheme are not actually drawn, to avoid Confusion.

Example VI.

Fig. 20. In this Scheme the Vanishing Line of the Ground, which the Piece of Building, &c. stand upon, is $AICKB$ passing through the Center of the Picture C , the Distance of the Picture being equal to CO . BG and AH are the Vanishing Lines of the upright Planes $abde, DE, N$, &c. and $abc, DFnm$, &c. G and H being the Vanishing Points of the Lines be , and mn , which touch the upper Corners of the two Flights of Steps, and consequently AG and BH are the Vanishing Lines of the Planes, that touch the upper or the under Edges of the Steps.

The given Side br of the Base of the regular Tetraedron is parallel to the Vanishing Line AB . u , which is the Projection of the Center of the Base brp , and the Seat of the Vertex o , is found by

by drawing pq parallel to AB , and Ir cutting it in q , and then drawing Cp and bq meeting in u . CL perpendicular to AB is the Vanishing Line of a Plane perpendicular to the Base kpr standing on up , and passing thro' the Line po , whose Vanishing Point is L , found (by *Prob. 11.*) by making CQ equal to the Distance of the Picture, and the Angle LQC equal to the Original of the Angle upo . VX is the Vanishing Line of the Face upr , by the help of which that Face is described, having given pr (by *Prob. 12.*) as the Face hrp was described on hr . The regular Octaedron, and Icosaedron, are described by their Ichnography and Orthography, which Method is sufficiently explain'd in the foregoing Example. I will only inform the Reader, that in the Orthography $ABCDEFGH$ of the Icosaedron, the Lines CD, AF, GH are equal, as also are AC, FD, BE , and AF is to BE , as the lesser Segment is to the greater of a Line cut in extream and mean Proportion.

The Light is supposed to come from the Sun, and the Rays are parallel to the Picture and to the Line AM ; so that the Shadow P of any Point D , is found by drawing NP through its Seat parallel to AB , and DP parallel to AM cutting it in L . For the Shadow of the Tetraedron, which is turned up on the End of the Steps; having in the same manner found s , (which would be the Shadow of o on the Ground, if the Steps were away) drawing sb and sp , are got the Shadows bo and po . Let su and sp cut the lower Edge of the Steps in y and x ; then drawing xt perpendicular to AC and cutting as in t , is got the
 Shadow

Shadow t of the Vertex o on the End of the Steps, and drawing xt is got that part of the Shadow of op , which falls on that End. And in the same manner is got the Shadow of ob . All the Rays being parallel to AM , and A being the Vanishing Point of DF , AM is the Vanishing Line of the Plane made by the Rays which pass thro' the Line DF , and BM being the Vanishing Line of the Wall, the Shadow Ff is cast on; M is the Vanishing Point of the common Intersection of those two Planes, that is, of the Shadow Ff of the Line DF .

Example VII.

Fig. 21. In this Scheme ACB passing through the Center of the Picture, C is the Vanishing Line of the Ground; A is the Vanishing Point of the Edge EF which the Beam rests upon, and the other Edges parallel to it; D is the Vanishing Point of the Edge GH , &c. and DB perpendicular to AB , is the Vanishing Line of the Plane EGH ; P and Q are the Vanishing Points of the Sides MK and KN , of the regular Tetraedron, and consequently PQ is the Vanishing Line of the Triangle KNM ; L is the Light and A its Seat on the Ground. Having BEg , that is the intersection of the upright Plane EGH with the Ground, and consequently g , where HG meets it, is the intersection of the Edge HG with the Ground. BE cuts the Edge SR of the Wall SRh in R , and consequently Rb perpendicular to AB is the intersection of the Wall with the Plane EGH , and consequently b , where Rb and GH meet, is the intersection of the Line GH

GH with the Wall. DL is the Projection of a Line parallel to the Original of GH (because of the Vanishing Point D) and B is its Seat, wherefore l is its intersection with the Ground.

Now the Originals of L and HG being parallel, they are in the same Plane, *viz.* in the Plane which makes the Shadow of HG; but lg represents the intersection of that Plane with the Ground; wherefore lg continued is part of the Shadow, and drawing LG cutting it in g , g is the Shadow of G, and gS is that part of the Shadow of HG which is on the Ground. Then drawing Sh and LH cutting it in h , hS is the other part of that Shadow against the Wall.

Having drawn gp and DT both parallel to AB, Dg and gp are the Projections of two Lines in a Plane, whose Vanishing Line is DT; and T is the Vanishing Point of the common intersection of that Plane with the Plane of the Triangle KMN (by Cor. 2. Th. 7.) PQT being its Vanishing Line. Therefore drawing Tp cutting gD in V, V is the intersection of the Line gGH with the Plane of that Triangle KMN. Then lg cutting MK in r , drawing rtV , rt is that part of the Shadow of GH, which falls on the Triangle KMN.

Having thus explain'd the manner of finding the Shadow of GH, the rest needs no Explanation.

Example VIII.

Fig. 22. In this Scheme C is the Center of the Picture, and CA the Vanishing Line of the Ground, and of the Surface of the Water which gives

gives the Reflexions; and S is the Vanishing Point of the Rays of Light, which are supposed to come from the Sun.

The Shadow of the perpendicular Line BD is found thus: SA drawn perpendicular to CA gives the Vanishing Point A of the Shadow on the Ground Bf. Then in the Circumference of the Base of the Cylinder (which is parallel to the Picture, its Axis being perpendicular to it, and consequently having the Vanishing Point C) taking any Point E, and finding its Seat on the Ground *e*, drawing CE, and Ce cutting BA in *f*, and then drawing *f*P perpendicular to CA and cutting CE in P, is got one Point P of the Shadow on the Surface of the Cylinder. And in the same manner are got all the other Points of that Shadow. To prove this, the Reader need only consider, that the Original of *eEPfe* is an upright Plane cutting the Cylinder in EP, and *fP* is the Shadow of BD on that Plane. Wherefore P is the Point where that Shadow falls on the Surface of the Cylinder. Any Point Q of the Shadow of the Circumference of the inward Cylinder on its Surface, is found thus. Having drawn CS, parallel to it is drawn at pleasure GH cutting that Circumference in G and H. Then drawing GS and CH meeting in Q, Q is the Point sought. For C being the Vanishing Point of the Axis of the Cylinder, as well as of CH, HQ is in the Surface of the Cylinder, and GH being parallel to CS, is the Projection of a Line parallel to the Picture, in a Plane whose Vanishing Line is CS (and its Original being parallel to the Picture, is in the Base of the Cylinder, which is parallel

parallel to the Picture.) Therefore the Originals of HG , HC , and GS being in the same Plane, Q is the Projection of the Point where the Ray of Light, whose Projection is GS , cuts the Surface of the Cylinder; that is, it is the Projection of the Shadow of the Original of the Point G in the Circumference of the Base, on the inward Surface of the Cylinder.

b being the Seat of the Point D on the Surface of the Water, the Reflexion d of the Point D is found by continuing the Perpendicular Db till bd is equal to bD . This is evident, because the known Law of Reflexions, is, that the Reflexions of all Objects appear to be as much on one Side of the reflecting Plane, as the real Objects are on the other Side of it. In AS , making As equal to AS , any Point q in the Shadow on the Surface of the inward Cylinder in the Reflexion, is found in the same manner as Q in the real Figure, using the Point s instead of S .

The Shadow of the Cylinder on the Surface of the Cone, is found by such another Expedient, as the Shadow of the Line BD on the Surface of the Cylinder.

Example IX.

Fig. 23. In this Scheme every thing else being easily to be understood by what has been already explain'd, I shall only shew the manner how the Reflexion is found in the Looking Glass of the Picture on the Eazle.

A is the Center of the Picture, and AB the Vanishing Line of the Ground; the Distance of the Picture being equal to AB . AC is the Vanishing

nishing Line of the Picture on the Eazle, and CD the Vanishing Line of the Looking Glafs.

Through a , where the Edge ba of the Leg of the Table cuts the Surface of it, drawing ae , and through b drawing bd , both parallel to AB , bd cutting the Intersection cd of the Surface of the Picture on the Eazle with the Ground in d , and then drawing de parallel to AC , and cutting ae in e , and then drawing Ae , is got the Projection Ae of the common Intersection of the Surface of the Table, and of the Picture on the Eazle. For ae being parallel to AB , is the Projection of a Line in the Surface of the Table parallel to the Picture, and for the same reason bd is the Projection of a Line on the Ground, and de is the Projection of a Line in the Plane of the Picture on the Eazle, both of them parallel to the Picture; ab is also the Projection of a Line parallel to the Picture. Therefore $abde$ is the Projection of a Trapezium parallel to the Picture, whose Angle e is in the common intersection of the Surface on the Table, and of the Picture on the Eazle. But A being the common intersection of the Vanishing Lines of those two Planes, is the Vanishing Point of their common intersection, and therefore eA is the Projection of that intersection (by *Cor. 2. Th. 7.*) For the same reason o being the Projection of the Point where the Surface of the Glafs touches the Table, and E being the common intersection of the Vanishing Lines AB and CD , oE is the Projection of the common intersection of the Surface of the Table and the Surface of the Glafs. Therefore f where oE and eA meet, is the Projection of the Point
where

where the three Planes meet, of the Surface of the Table, the Glass, and the Picture on the Eazle. Therefore drawing fC , it is the Projection of the common intersection of the Picture on the Eazle and the Looking Glass.

Having found the Vanishing Point P of Lines perpendicular to the Plane of the Looking Glass, whose Vanishing Line is CD (by *Prob. 14.*) drawing PA thro' the Vanishing Point A of the Line GH , and cutting CD in D , D is the Vanishing Point of the Seat of GH on the Plane of the Glass. Therefore GH cutting Cf in i , Di is the Projection of that Seat. Then drawing GP cutting Di in k , k is the Seat of the Point G on the Glass. Wherefore in GP making kg to represent a Line equal to that represented by Gk (by *Prob. 3.*) g is the Projection of the Reflection of G , and gi is the Reflexion of Gi , and drawing PH cutting gi in h , gh is the Reflexion of GH . And in the same manner may be found any other Lines in the Reflexion.

The Reflexion of the Picture on the Eazle may also be described by its Vanishing Line, in the same manner as the Projection of the Picture it self was described; for in PAD making a D to represent a Line equal to that represented by AD , a is the Vanishing Point of the reflected Line gh , and Ca is the Vanishing Line of the reflected Picture on the Eazle.



LINEAR PERSPECTIVE.



PART II.

Of the Manner of finding the Original Figures from their Projections given, and of the Situation that is necessary to be observed for viewing particular Projections.



PROBLEM XVIII.

Having given the Projection of a Line divided, and its Vanishing Point; to find the Proportion of the Parts of the Original.



LET AB (Fig. 7.) be the given Projection, divided in C, and V its Vanishing Point. Draw at pleasure VO, and *ab* parallel to it, and from any Point O in the Line OV draw OA, OB, OC, cutting *ab* in *a*, *b*, and *c*. Then will

will the Original of AC be to the Original of CB, as *ac* is to *cb*.

COROL. $ac : cb :: AC \times BV : BC \times AV$.

PROBLEM XIX.

Having given the Projection of a Line divided into two Parts, and the Proportion of the Original; to find its Vanishing Point.

Let AB (*Fig. 17.*) be the Projection given divided in C. Through C draw at pleasure a Cb, and in it make a C to Cb, as the Original of AC is to the Original of CB, and draw a A and bB meeting in O. Parallel to ab draw OV cutting AB in V, which will be the Vanishing Point sought.

COROL. $BV : BA :: Ca \times CB : Cb \times AC$
 $= Ca \times CB$.

These two last Problems, with their Corollaries, follow easily from *Prob. 3.* and its *Cor.*

PROBLEM XX.

Having given the Projection of a Triangle, with its Vanishing Line, its Center and Distance; to find the Species of the Original Triangle.

Let *abc* (*Fig. 10.*) be the Projection given, HG its Vanishing Line, and S its Center, and SO perpendicular to HG, and equal to its Distance. Having continued the Sides of the Projection given,

given, till they cut the Vanishing Line in their Vanishing Points G, H, I, draw GO, GH, and GI, and the Originals of the Angles bac , abH , acb , will be equal to GOI, IOH, GOH, respectively (by *Prob. 11.*) Whence the Species of the Original Triangle is given.

PROBLEM XXI.

Having given the Projection of a Triangle of a given Species, and its Vanishing Line; to find the Center and Distance of that Vanishing Line.

Let ABC (*Fig. 12.*) be the given Projection, and FD its Vanishing Line. Continue the Sides of the Projection till they cut the Vanishing Line in their Vanishing Points D, E, F. Bisect DE and EF in G and H, and draw GI and HK perpendicular to FD, making GI to GE as Radius is to the Tangent of the Angle represented by BAC, and KH to EH as Radius is to the Tangent of the Angle represented by BCA; so that EIG and FKH may be equal to those Angles. With the Centers I and K and the Radius's IE and KE, describe two Circles cutting each other in O, and draw OS cutting FD at Right Angles in S. Then will S be the Center, and SO the Distance sought.

DEMONSTRATION.

Supposing S to be the Center and SO the Distance of the Vanishing Line FD, the Originals of the Angles BAC and BCA will be equal to DOE, and EOF (by *Prob. 11.*) But by the nature

nature of the Circle DOE and FOE are equal to GIE and HKE, which by the Construction are equal to the Angles that ought to be represented by BAC and BCA. Therefore S is the Center and SO the Distance sought.

PROBLEM XXII.

Having given the Projection of a Trapezium of a given Species; to find its Vanishing Line, Center and Distance.

Let $abcd$ (Fig. 12.) be the Projection given. Draw the Diagonals ac , bd , meeting in e , and by the Proportions of the Originals of ae , ec , and be , ed , find the Vanishing Points E and F, of the Lines ac and bd (by Prob. 19.) Draw FE, which will be the Vanishing Line sought. Then by the given Species of the Original of the Triangle abe , find the Center S and Distance SO (by Prob. 21.)

PROBLEM XXIII

Having given the Projection of a Right-angled Parallelopiped; to find the Center and Distance of the Picture, and the Species of the Original Figure.

Let ABCDEFG (Fig. 24.) be the Projection given. Continue the Projections of the parallel Sides, till they meet in their Vanishing Points H, I, K, and draw HI, HK, IK, which will be the Vanishing Lines of the several Faces of the Figure sought,

sought, containing a solid Right Angle. Draw KL perpendicular to HI, and HM perpendicular to KI meeting in S; which will be the Center of the Picture (by *Cor. 2. Prob. 16.*) Then on the Diameter LK describe a Circle, and draw SO perpendicular to LK cutting it in O, and OS will be the Distance of the Picture (by *Prob. 14.* LOK being a Right Angle upon account of the Circle.) Lastly, find the Distances of the Vanishing Lines KI and IH (by *Cor. 3. Prob. 16.* M and L being their Centers, by *Th. 1.*) and then find the Species of the Originals of the Faces DAFE, and DABC (by *Prob. 20.*)

COROL. When the Vanishing Line of one of the Faces (suppose IH) passes through the Center of the Picture, the Vanishing Point K of the Sides perpendicular to it, will be at an infinite distance: by which means the Situation of LK will be indetermin'd. So that the Species of that Face ABCD may be taken at pleasure, and then the Center and Distance of the Picture may be found by *Prob. 20.* And in this case, if it were only required that the Projection proposed should represent a right angled Parallelopiped in general, the Place of the Point of Sight might be any where in the Circumference of a Circle described on the Diameter HI, and in a Plane perpendicular to the Picture. This I leave as a hint that may be useful to the Painters of Scenes in Theaters.



F I N I S.



APPENDIX.



NUMB. I.

The Description of a Method, by which the Representations of Figures may be drawn on any Surface, be it never so irregular.

FROM what has been said in this Book to explain the Principles of Painting, especially at the End of the Definitions, it is evident that the Sense of the 2^d Theorem may be extended to any Surface that Figures are painted on, be it Concave or Convex, or never so irregular. So that be the Surface of the Picture ABC (*Fig 3.*) of any Form whatsoever, the Projection *fg* of the Original Line FG, will still be the intersection of the Picture with the Plane of the Triangle FGO. But the Parallel OV is in that Plane (*Th. 3.*) Hence it follows, that if the

K

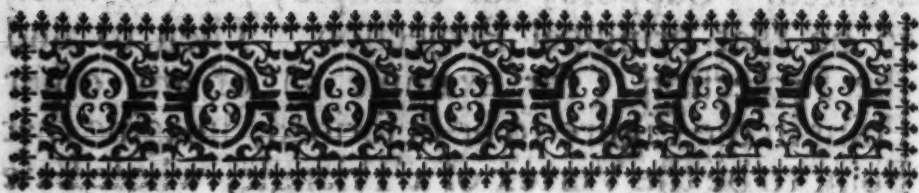
Flame

Flame of a Lamp be so placed, as to cast the Shadow of OV on any Point B of the Line FG , it will cover the whole Line FG , and at the same time cover the Line $VfgB$ on the Picture, which is the Projection of FG ; all the Rays coming from the Lamp, and passing by the Line OV , in that case making a Plane, which coincides with the Plane of the Triangle OFG . The same thing will happen, if a Person should place his Eye so as to make any Point of the Line FG to seem to be cover'd by the Line OV ; in that case OV will seem to cover all the Projection Vfg . So that if a Lamp be so placed, as to make the Shadow of OV to pass thro' any one Point of the Projection Vg , it will coincide with the whole; and if a Person places himself so as to make OV to seem to cover any one Point of the same Projection, it will seem to cover the whole. Hence I imagine, that the following Method may be of use for drawing the Projections of any Figures on any Surface, suppose on the Walls and Cupola's of Churches; the Walls and Cielings of great Rooms, the Scenes of Theaters, &c.

Chuse some principal Line in the Design to be drawn, and having by some proper Method found the Projections of its extream Points, through the Point of Sight pass a Thread parallel to the Original Line, and cast the Shadow of it on those two Points already found, and that Shadow mark'd with a Crion will be the Projection of that principal Line; or, if in any particular case it happens to be more convenient, place your Eye so as to make that parallel Thread to seem to cover those two Points already mark'd, and instruct

instruct an Assistant to mark out the Projection wanted. Suppose, for example, *ph* (Fig. 20.) to be that principal Projection. Then to find the Projection (for example *o*) of any other Point in the Figure to be described, imagine that Point to be the Vertex of a Triangle, whose Base is the Original of the Projection already found, and place the Thread (passing still through the Point of Sight) in a parallel Situation to the Original of one of the Legs of that Triangle, and cast its Shadow on the proper Extremity of the Projection given, and mark it as before, and you will have the indefinite Projection of that Leg, (suppose it to be *bo*.) Do the same by the other Leg, and by the intersection of those Projections (suppose of *bo* and *po*) you will have the Projection of the Point sought. And by this Method may be found the Projections of any Figures whatsoever. I shall not enlarge upon this Method, not having had an opportunity of putting it in practice; for which reason I only propose it as a Hint, which I leave to be further consider'd of by the Curious.





NUMB. II.

*A New THEORY for mixing of COLOURS,
taken from Sir Isaac Newton's Opticks.*

THOU' my Design was only to treat of *Linear Perspective*, and (not to discourse of all the Parts of *Painting*, yet as this Book will be most useful to those who practise that Art, I thought it would not be improper, nor unenter-
taining to the Readers, if I took this Occasion to publish some Thoughts I have had concerning the Mixture of Colours: which I have fallen into upon considering Sir *Isaac Newton's* Theory of Light and Colours, in his most excellent Treatise of *Optics*.

In Colours these two things are to be consider'd, the Hue, (which is properly what may be called the Colour,) and the Strength of Light and Shadow. For as different Colours, suppose Red and Green, may have the same Strength of Light; so two things, that are one of them much darker than the other, may still have the same Hue, as a light Blue and a dark Blue,

With

With respect to the Hue, these two things are to be consider'd, 1st. The Species of Colour, and 2^{dly}. The Perfection and Imperfection of Colour under the same Species. Colours differ in Species, as Blue and Red, and Colours of the same Species differ in degree of Perfection; as the Red of a Field Popy is much more perfect than the Red of a Brick. This Quality of Perfection and Imperfection in the Colours, by the Painters is express'd by the Terms Bright, or Clean, or Simple, and Broken; which is taken from their Method of making the imperfect Colours, by the Mixture of other Colours, which is called breaking the Colours. With respect to this Quality of Colours, Sir *Isaac Newton*, in the Book already mention'd, shews, that every Ray of Light has its proper Colour, which it always carries with it, and never loses, in whatever manner it happens to be reflected or refracted. These natural Colours of the Rays are the Bright Simple Colours, and the natural Order of them, as they appear when they are separated by the Refraction of a Prism, is, Red, Orange, Yellow, Green, Blue, Indico, Violet. All the less perfect or broken Colours, are made by the Composition and Mixture of these simple Colours, as Yellow Rays mix'd with Blue Rays, make a Green, but not so perfect as the simple natural Rays that are Green; and Red and Yellow Rays make an Orange Colour, but not so perfect as the natural Orange-colour'd Rays. And by a just Proportion of all the Natural Rays together, is produced Whiteness, which is indifferent to all the simple

simple Colours, and can't be said to incline more to one Colour than to another. By White I mean any Colour between the lightest White and the darkest Black ; for as we are not now considering the Degrees of Light and Shade, all the Colours from Black to White are to be consider'd as of the same Hue.

According to this Observation of the Nature of Whiteness, it appears that the broken Colours are a Medium between the simple Colours and White, and the more broken a Colour is, the nearer it is to White, and the further it is from White, the more simple it is.

Having thus explain'd the Nature of the Colours, and the Effect of their Mixture, in order to find exactly what Colour will be produced by the Mixture of any Colours given, Sir *Isaac* disposes the Colours in the following manner. Let there be a Circle made A D F A, and let the Circumference be divided into seven Parts AB, BC, CD, DE, EF, FG, G A, in the same proportion to one another as the Fractions $\frac{1}{2}, \frac{1}{12}, \frac{1}{12}, \frac{1}{2}, \frac{1}{12}, \frac{1}{12}, \frac{1}{2}$; which are the Proportions of the Musical Notes *Sol, la, fa, sol, la, mi, fa, sol*. Between A and B place all the Kinds of Red, from B to C place all the Kinds of Orange, from C to D place all the Kinds of Yellow, from D to E place all the Kinds of Green, from E to F place all the Kinds of Blue, from F to G place all the Kinds of Indico, and from G to A place all the Kinds of Violet. Having thus disposed the simple Colours, the Center of the Circle O will be the Place of White. And between the Center and the Circumference
are

are the Places of all the broken compounded Colours, those nearest the Center being the most compounded, and those farthest from it being the least compounded. As in the Line O_1 , all the Colours at 1, 2, 3, 4, are of the same Species, that is, Green inclining towards Blue, but the Colour at 1 is the simple natural Colour; that at 2 is something compounded, or broken; that at 3 is more broken; and that at 4 is still more broken.

The Colours being thus disposed, to know what Colour results from the Mixture of any Colours given, find the Center of Gravity of the Places of the Colours given, and that will shew the Character of the Compound. For example, suppose I would know what Colour would result from the Mixture of two Parts of the simple Yellow at P, with three Parts of the simple Blue at Q: I find the Center of Gravity 3 of the Points P and Q; that is, I draw PQ, and having divided it into five Parts (which is the Sum of three and two) I take the Point 3 three Parts from P (because there are three Parts of Blue) and two Parts from Q, (because there are two Parts of the Colour at P.) Then drawing O_3 cutting the Circumference in 1, by the Place of the Point 1, (which is between D and E, but nearer to E) I find the Mixture is a Green inclining towards Blue; but because 3 is near the Middle between the Center and the Circumference, the Colour is pretty much broken. To make the same thing more clear by another Example; suppose I would know what would result from a Mixture of two
Parts

Parts Yellow at P, three Parts Blue at Q, and five Parts Red at R. First I find the Place 3 of the Mixture of the Yellow and the Blue, as before. Then drawing the Line 3 R (because there are five Parts of the Colour at 3, and five Parts of the Colour at R) I divide it into ten Parts, and take the Point *r* five Parts distant from R. By this means *r* is the Center of Gravity of the three Colours at P, Q, and R, and is consequently the Place of the Mixture; which by drawing Or cutting the Circumference in *s*, I find to be an Orange a little inclining towards Red, and because *r* is much nearer the Center than the Circumference, the Colour is very much broken. And thus one may proceed in other Cases.

Again, having given the Place of any compound Colour, one may find what Colours may be mix'd to compound it. Thus having given the Colour at 3, drawing any Line P 3 Q thro' 3, the Colour proposed may be made by a Mixture of the Colours in P and Q, taking such a Proportion of them as is expressed by the Lines 3 P and 3 Q, that is, taking of the Colour P as much as in proportion to 3 Q, and as much of the Colour Q as is in proportion to 3 P. Or having drawn O 3 passing thro' the Points 1, 2, 4, the same Colour may be produced by mixing the Colours in 1 and 4 in proportion to the Lines 4.3 and 2.3; or it may be produced by breaking the simple Colour at 1 with White (which is at O) in the proportion of the Lines 3.1 and 3 O. And thus in other Cases.

The Proportions hitherto mention'd of the Colours to be used in the Mixtures, relate to the Quantity of the Rays of Light, and not to the Materials which artificial Colours are made of. Wherefore if several artificial Colours were to be mix'd according to these Rules, and some of them are darker than others, there must be a greater Proportion used of the darker Materials, to produce the Hue proposed, because they reflect fewer Rays of Light in proportion to their Quantities; and a lesser Proportion must be used of the lighter Materials, because they reflect a greater Quantity of Light.

If the Nature of the material Colours, which are used in Painting, was so perfectly known, as that one could tell exactly what Species of Colour, how perfect, and what degree of Light and Shade each Material has with respect to its Quantity, by these Rules one might exactly produce any Colour proposed, by mixing the several Materials in their just Proportions. But tho' these Particulars cannot be known to sufficient Exactness for this purpose, besides the Tedioufness that would be in Practice, to measure the Colours according to their exact Proportions; yet the Knowledge of this Theory may be of great use in Painting. Suppose, for example, I had a Palate provided with the several Colours at *a, b, c, d, e*: Suppose for instance at *a* Carmine; at *b* Orpiment; at *c* Pink; at *d* Ultramarine, at *e* Smalts; and I had occasion to make a broken Green, such as I judge should be placed at *x*. Looking round the Point *x*, I see that it does not lie a great
L deal

deal out of a Line drawn thro' c and d ; therefore I conclude, that mixing the Colours c and d will come very near what I want. But because x is nearer to the Center O than the Line cd , having brought my Tint as near as I can to what I want, suppose to z , I look from z cross x for some Colour opposite to z , to break the Tint with, and I find the nearest to be a , therefore by mixing of the Colour a I bring the Composition to the Tint I have occasion for. If the Colour a carries the Tint too much towards the Line OD , I put a little more of the Colour d , which brings it into the right place. Or having got the Tint z , I might have broken it with White, whose place is at the Center O . Or putting a greater proportion of the Colour d instead of a , I may afterwards break the Tint by means of the Colour b . And in the same manner by only inspecting this Scheme, one may see in what manner to make any Tints whatsoever, that can be produced by the Colours that one uses. Thus one sees that Red and Yellow makes a broken Orange Colour, which may still be more broken by adding Blue, or Indico, or Violet, which are to be taken one or other, as one would have the Tint inclined more to the Yellow or to the Red; Blue bringing it toward the Yellow, and breaking it much; and Violet carrying it towards the Red, and not breaking it so much.

From these Principles one may see the Reason why the Materials of the brightest and simplest Colours are the most valuable, and of them why the lightest are most to be esteem'd. The simplest Colours

Colours are the most valuable, because they cannot be produced by Mixture; for Mixture always breaks the Colours. Suppose *a, b, c, d, e*, to be all the Colours you have, then drawing Lines to join the Points *a, b, c, d, e*, all the Tints that can be produced by those Colours will have their Places within the Area of the Polygone *abcde*. That the lighter Colours are more valuable than the dark ones, is because Black does not break the Colours so much as White; so that it is easier to make the clean dark Tints with light Colours and Black, than to make the bright light ones, with dark Colours and White. For by what has been shew'd, White breaks the Colours very much, but Black being nothing but the absence of Light, only darkens the Colours. Tho' upon account of the Imperfection of the Materials that are in use, Black does also break the Colours something, because there is no Material so perfectly black as to have no Colour at all, as one may see by the best Blacks having Lights and Shades. There will be other Exceptions also to be made in the application of these Observations to Practice, upon account of the particular Qualities of the Materials some Colours are made of. If all the Colours were as dry Powders, which have no effect upon one another, when mix'd, these Observations would exactly take place in the mixing of them. But some Colours are of such a Nature, that they produce a very different effect upon their Mixture, to what one would expect from these Principles. So that it is possible there may be some dark Materials, which when

when diluted with White, may produce cleaner and less compounded Colours than they gave when single; as some Colours do very well to glaze with, which don't look well laid on in a Body. But these Properties of particular Materials I leave to be consider'd by the Practitioners in this Art.



Fig. 2.

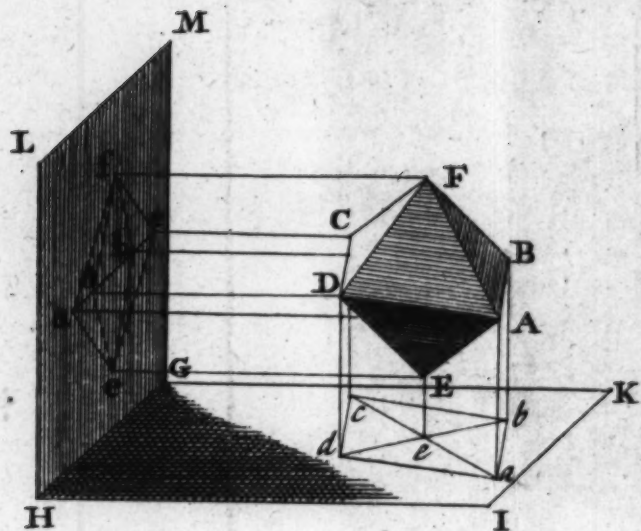


Fig. 1.

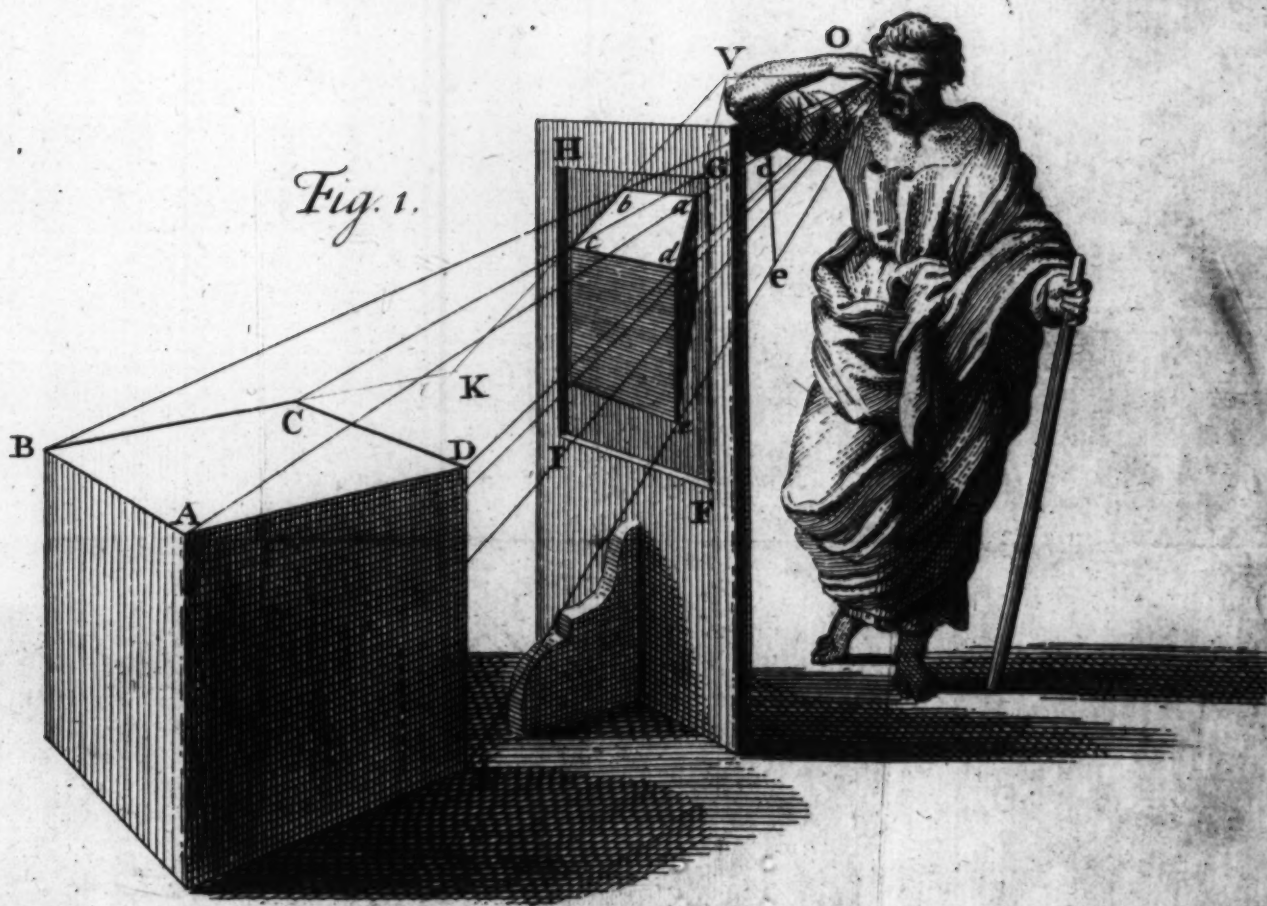


Fig: 3.

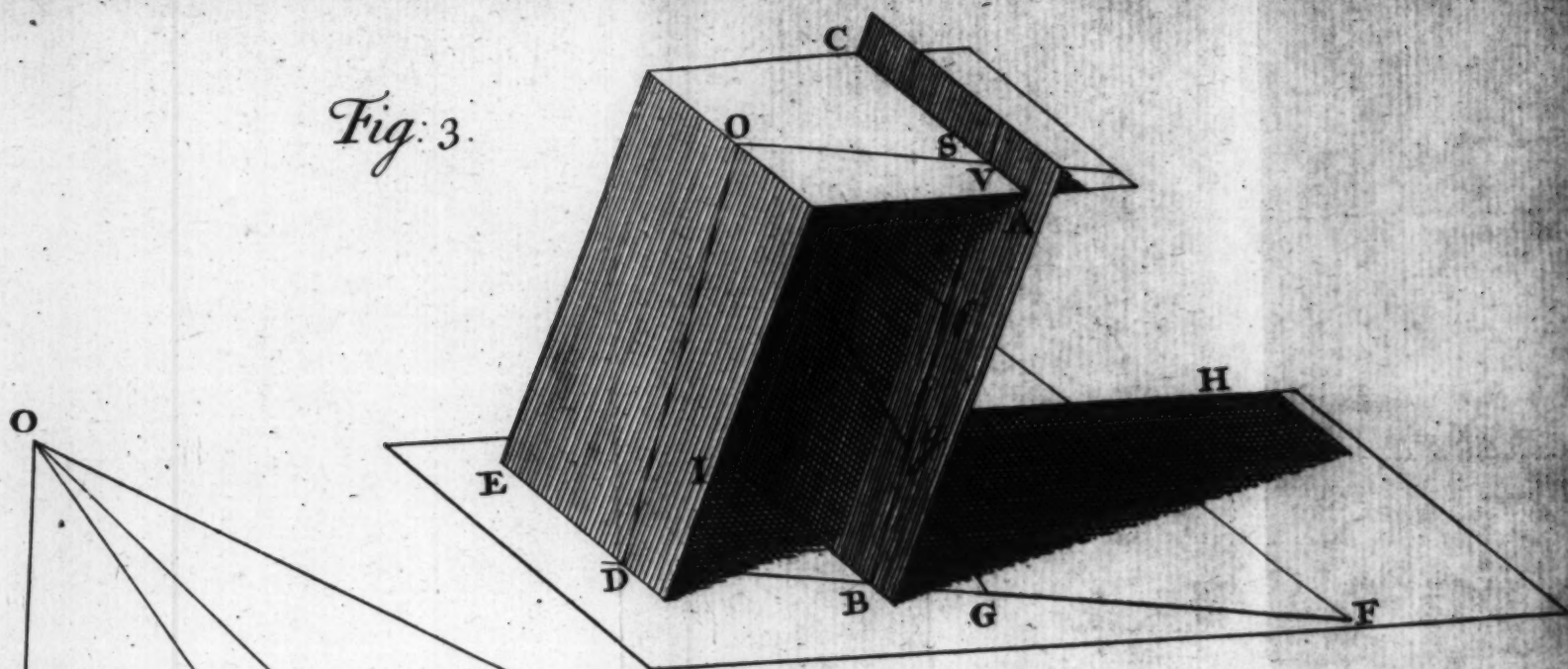


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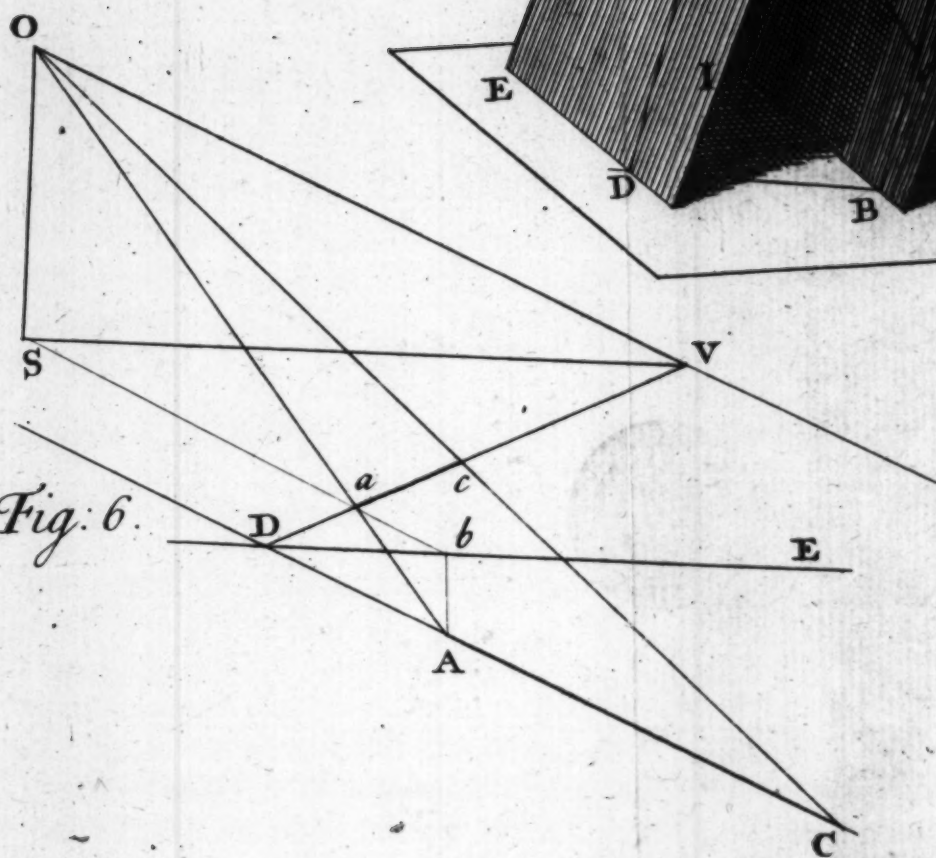


Fig: 4.

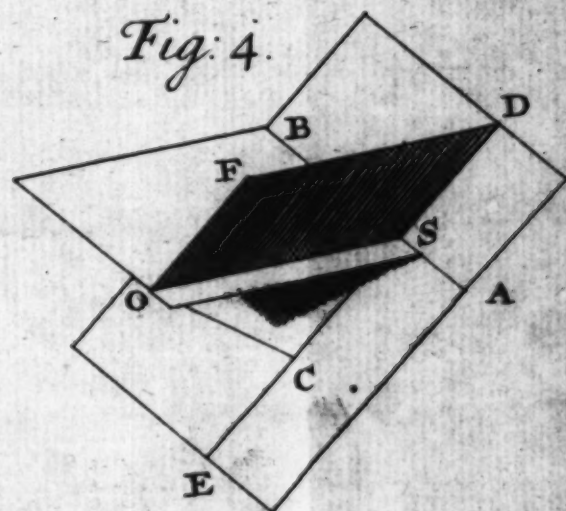
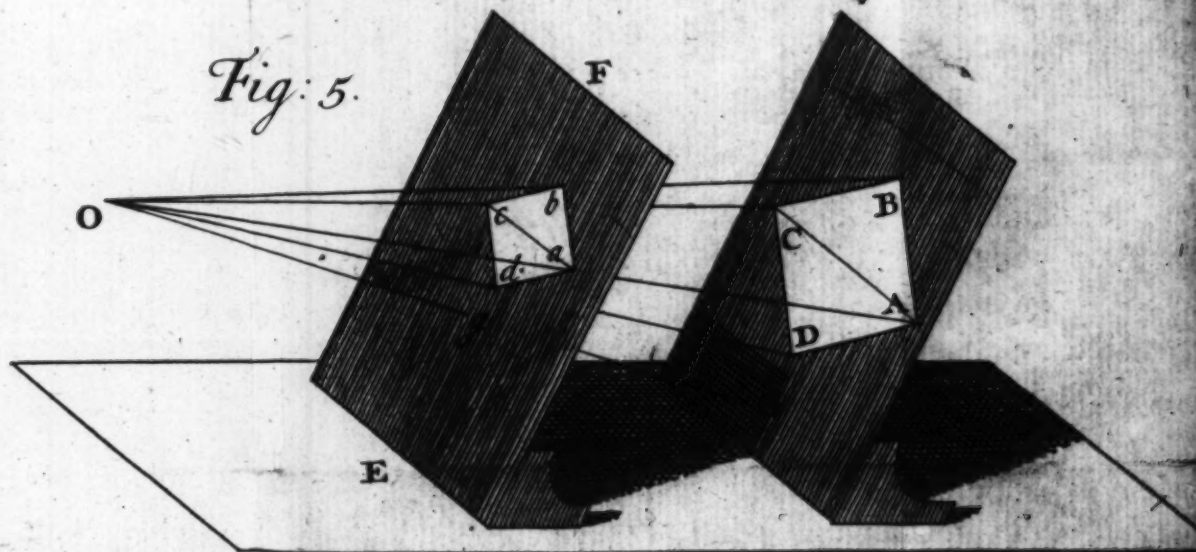


Fig: 5.



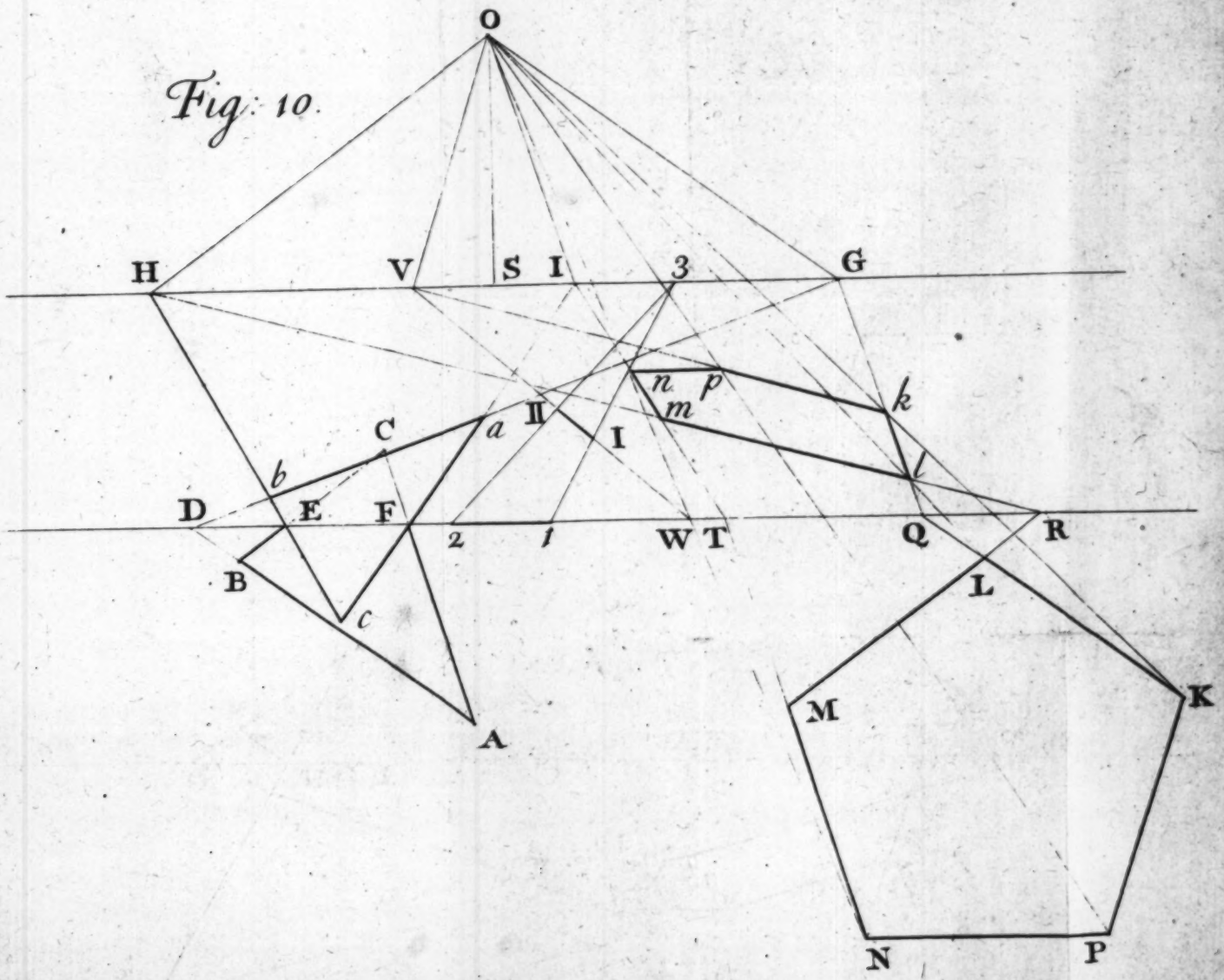
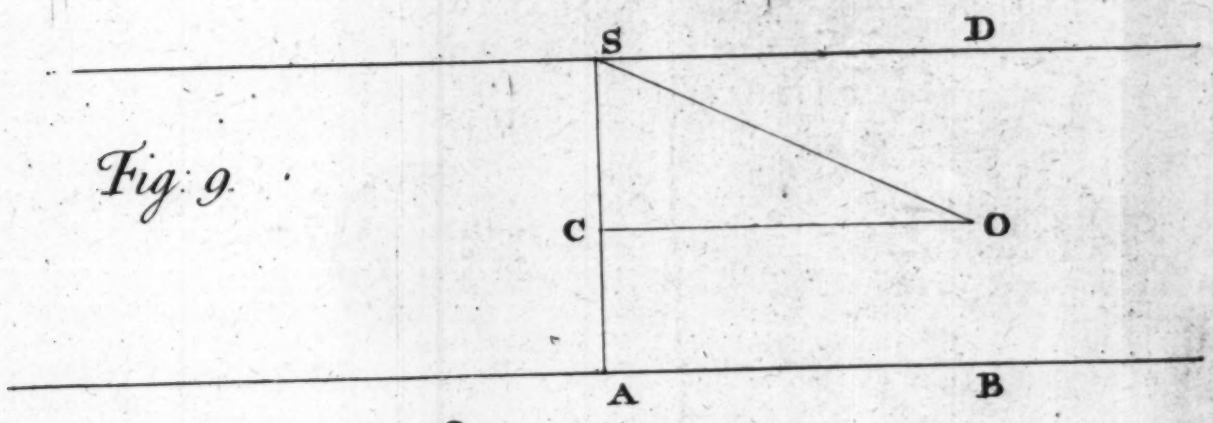
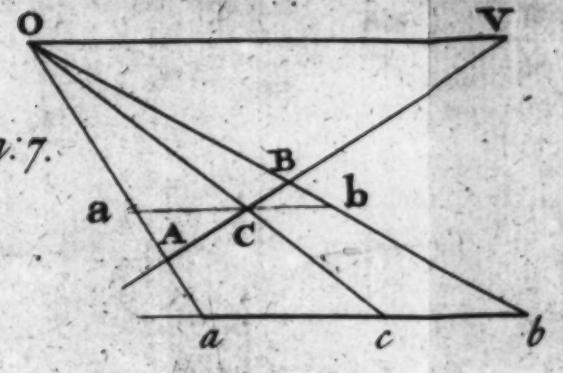
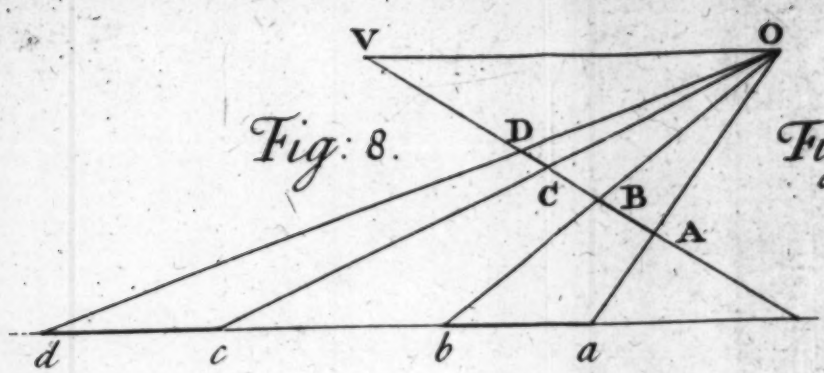


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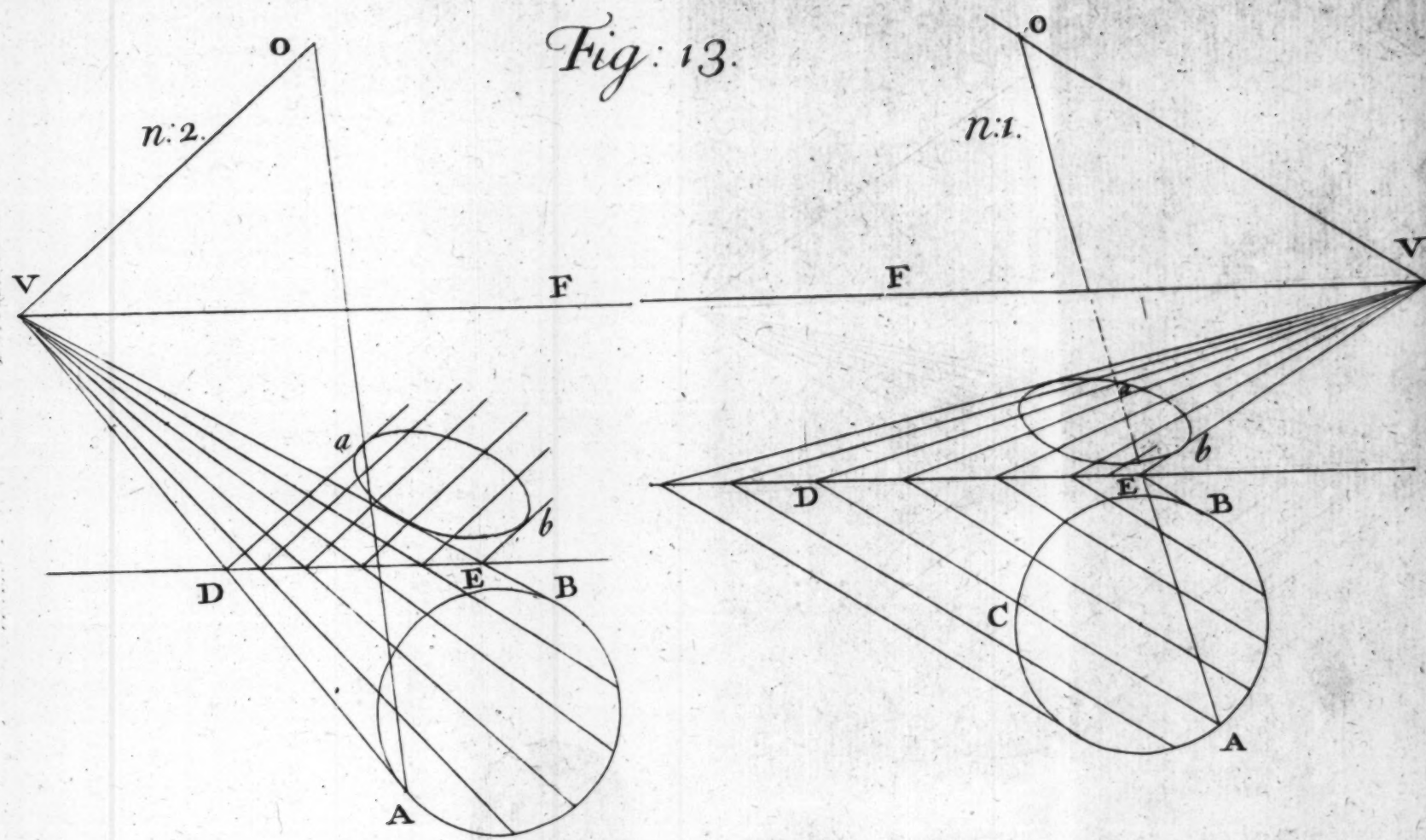
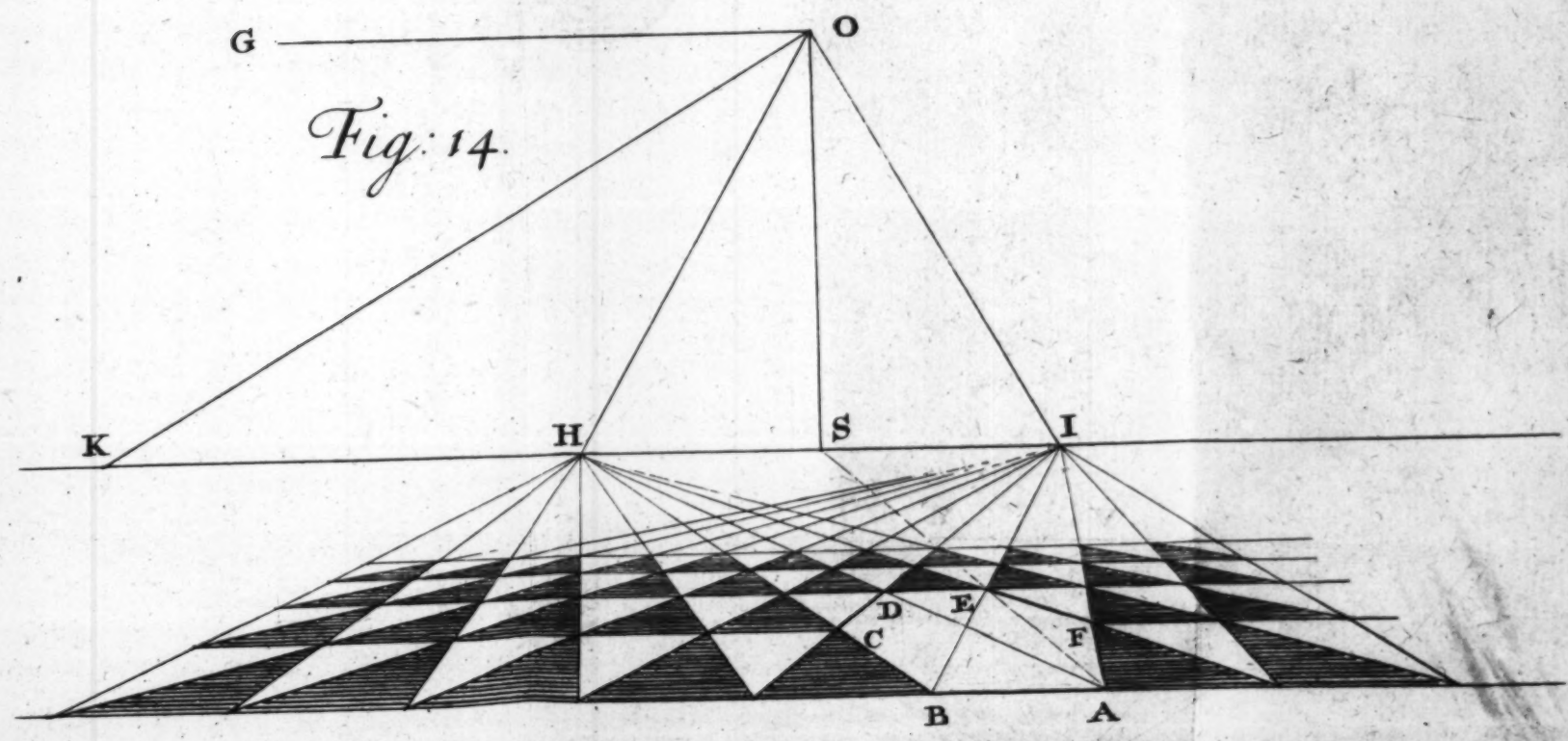


Fig: 14.



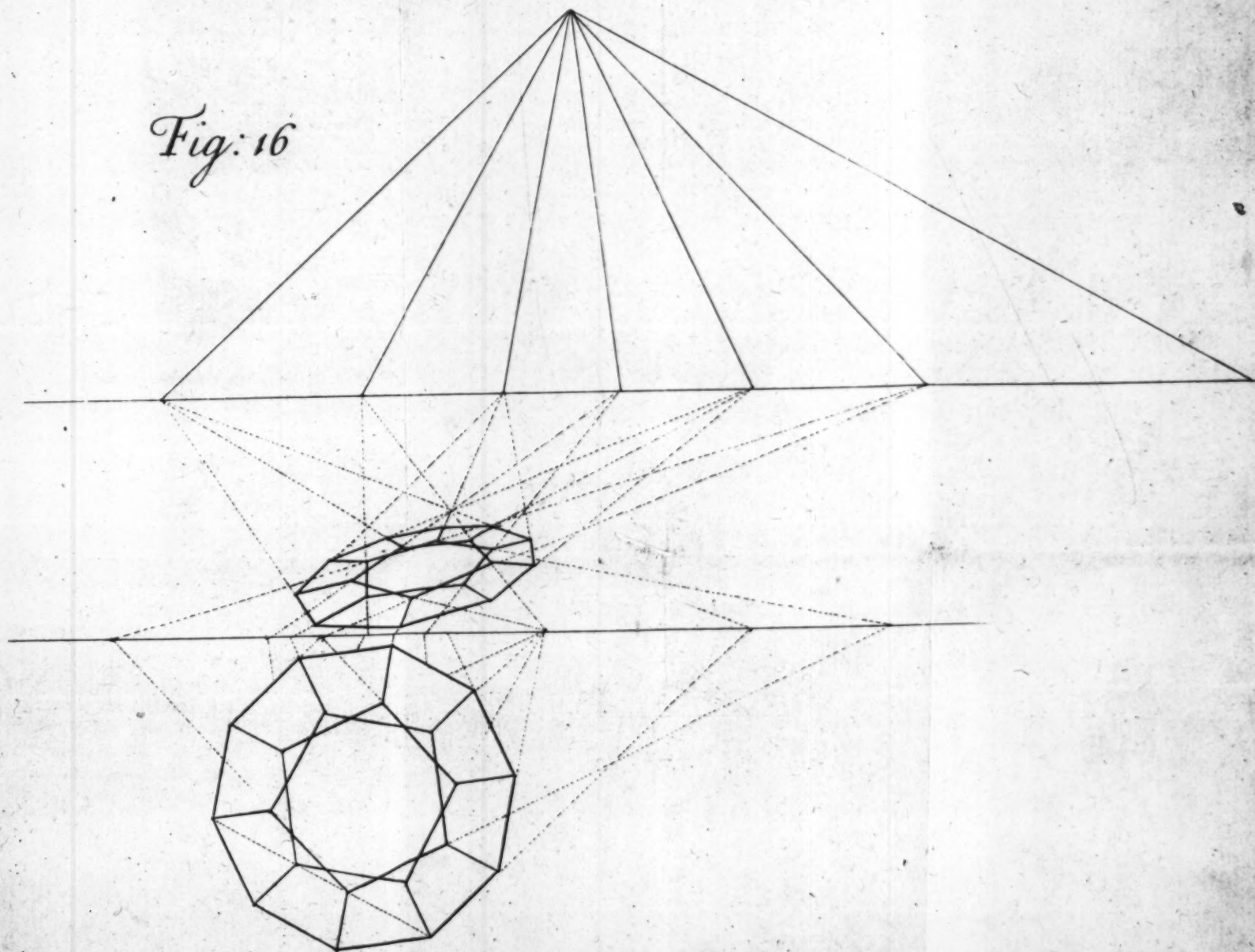
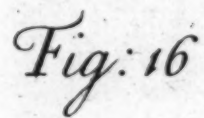
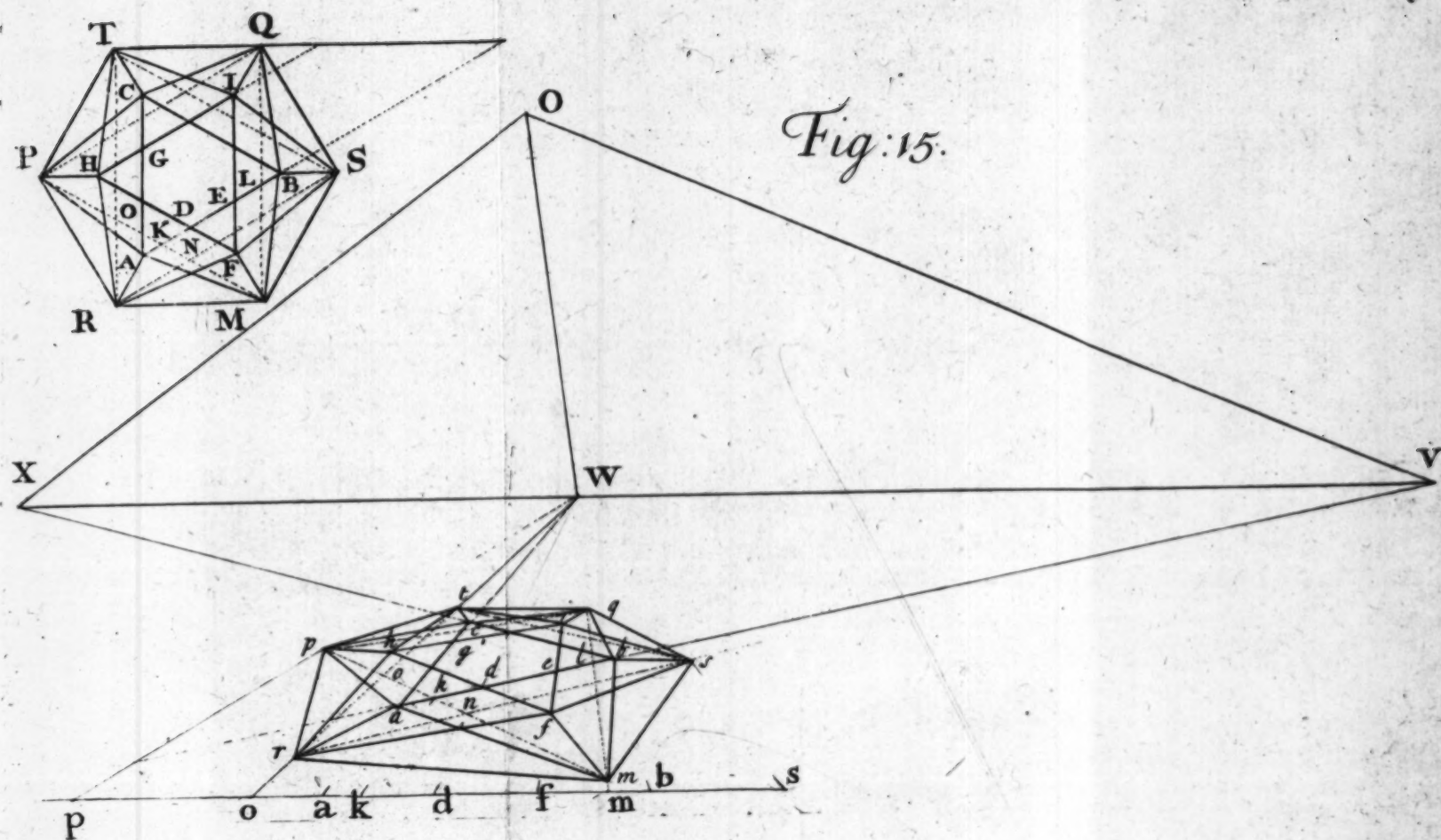


Fig: 17.

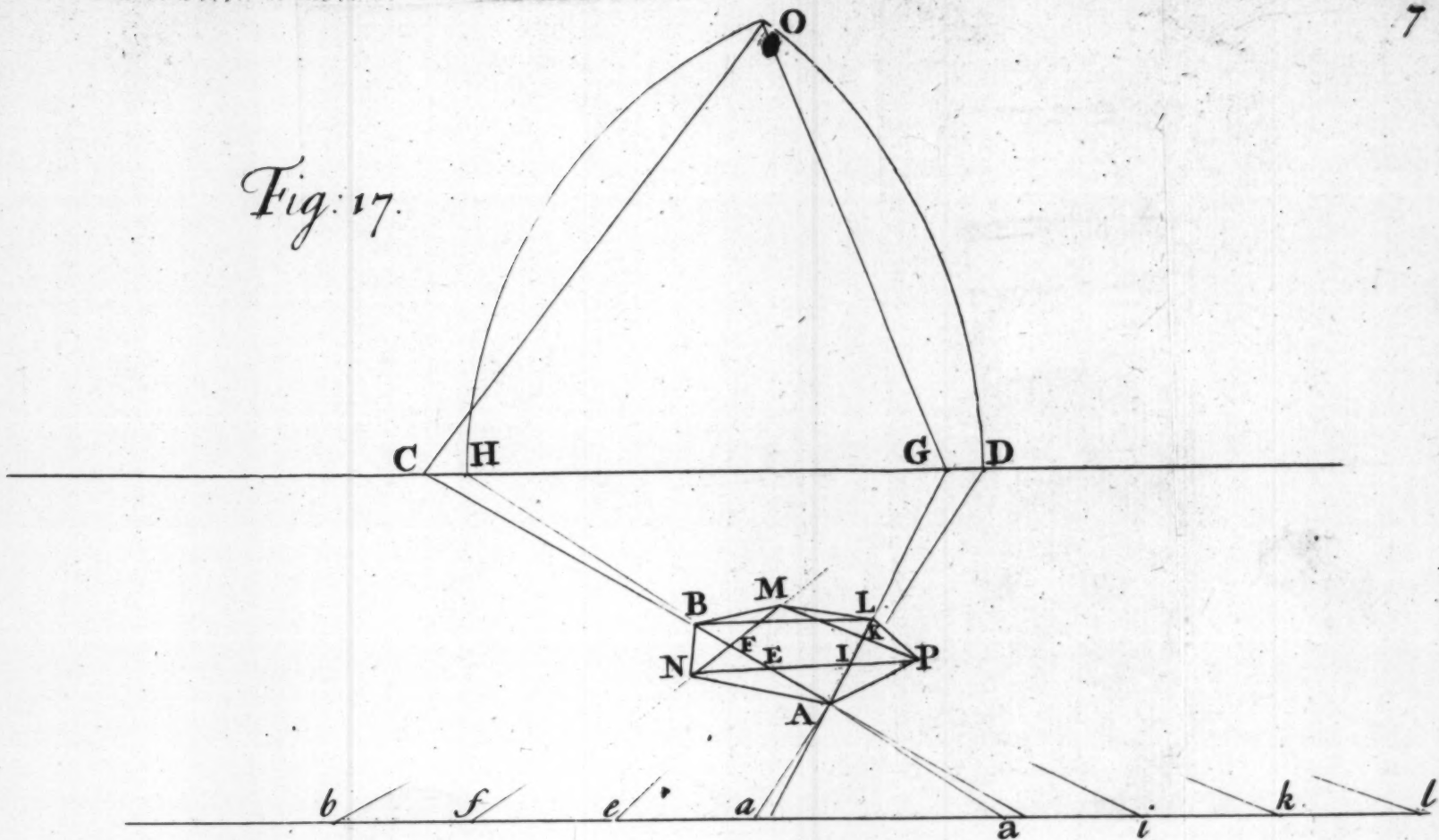
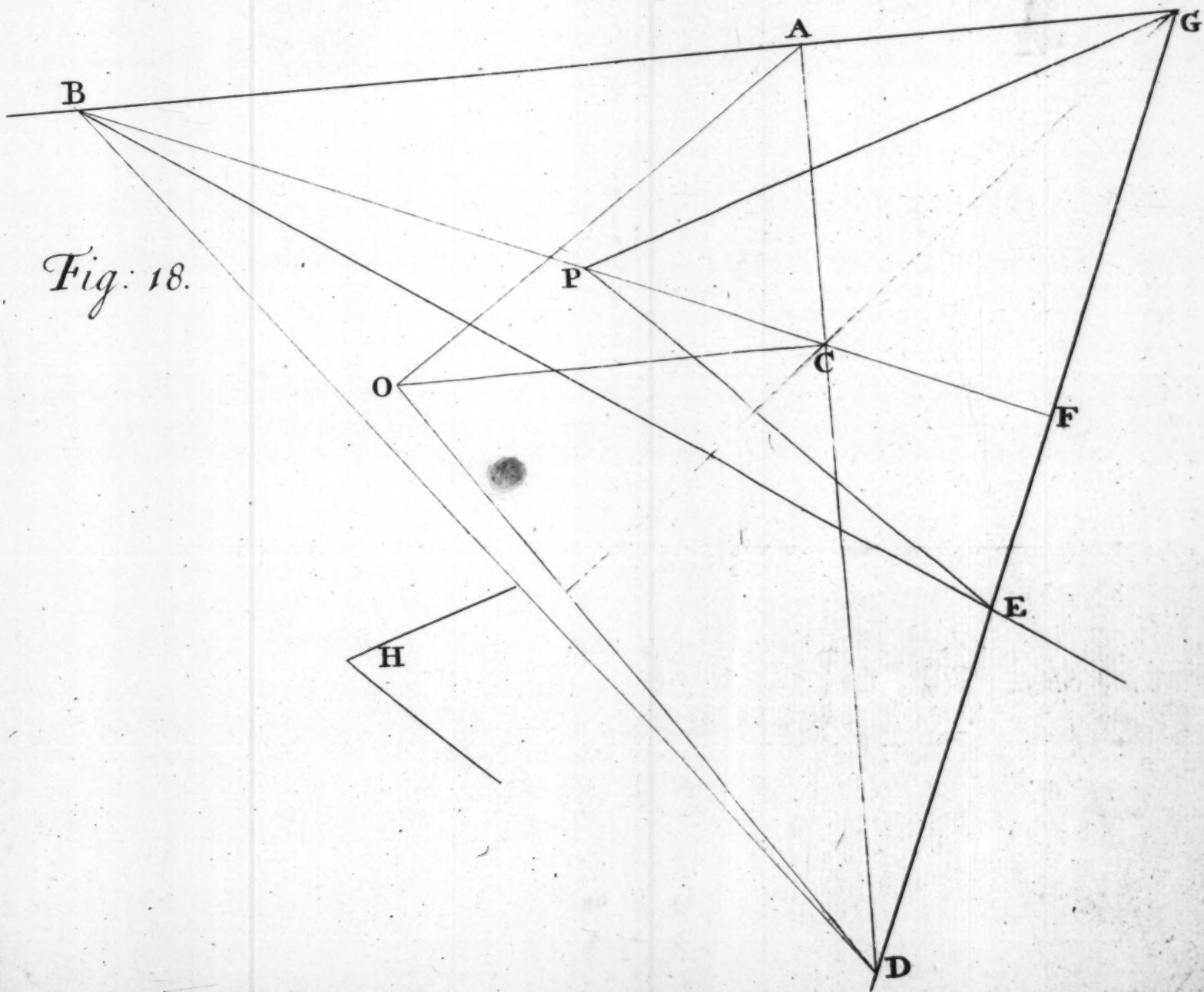


Fig: 18.



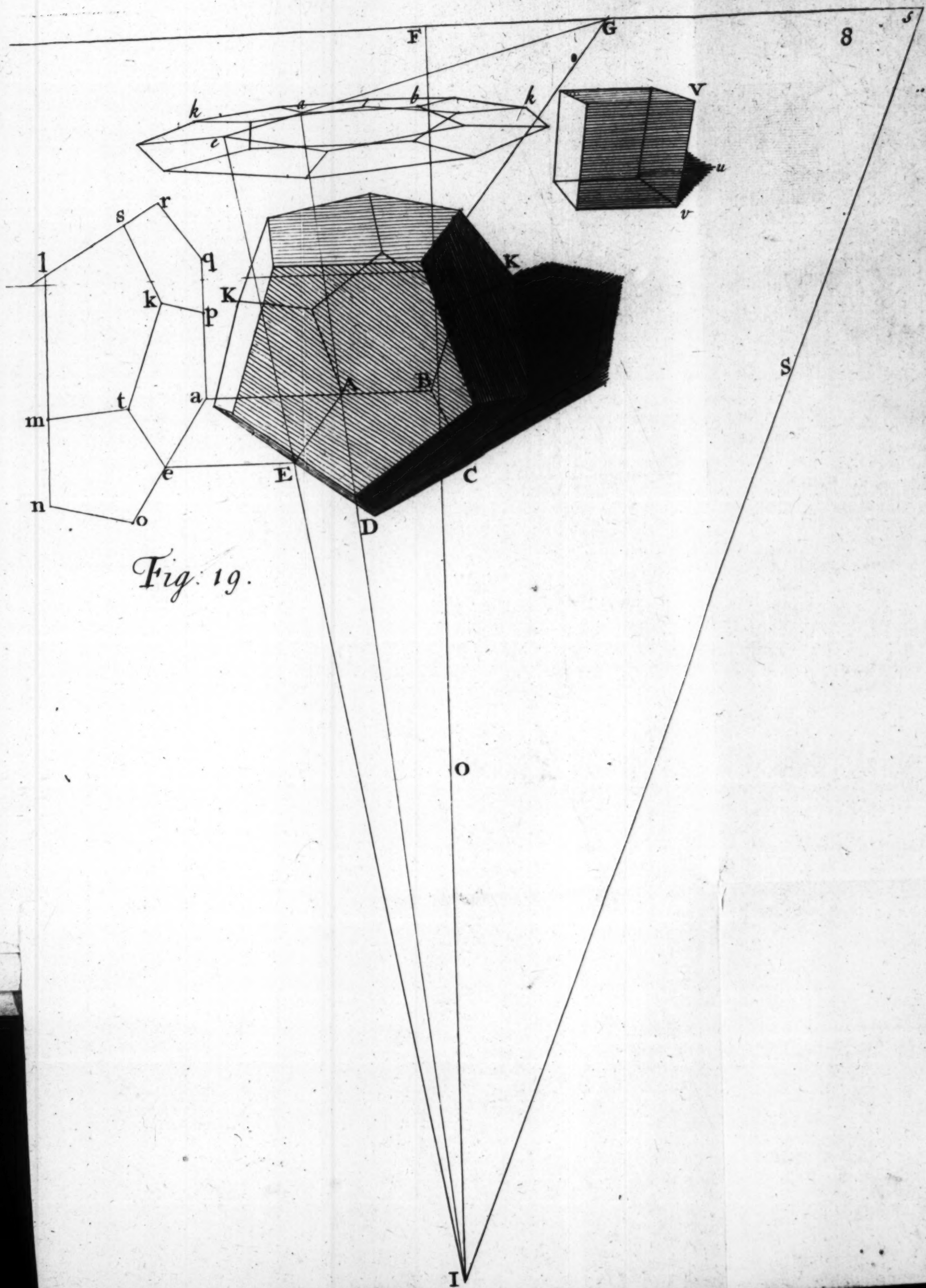


Fig: 21.

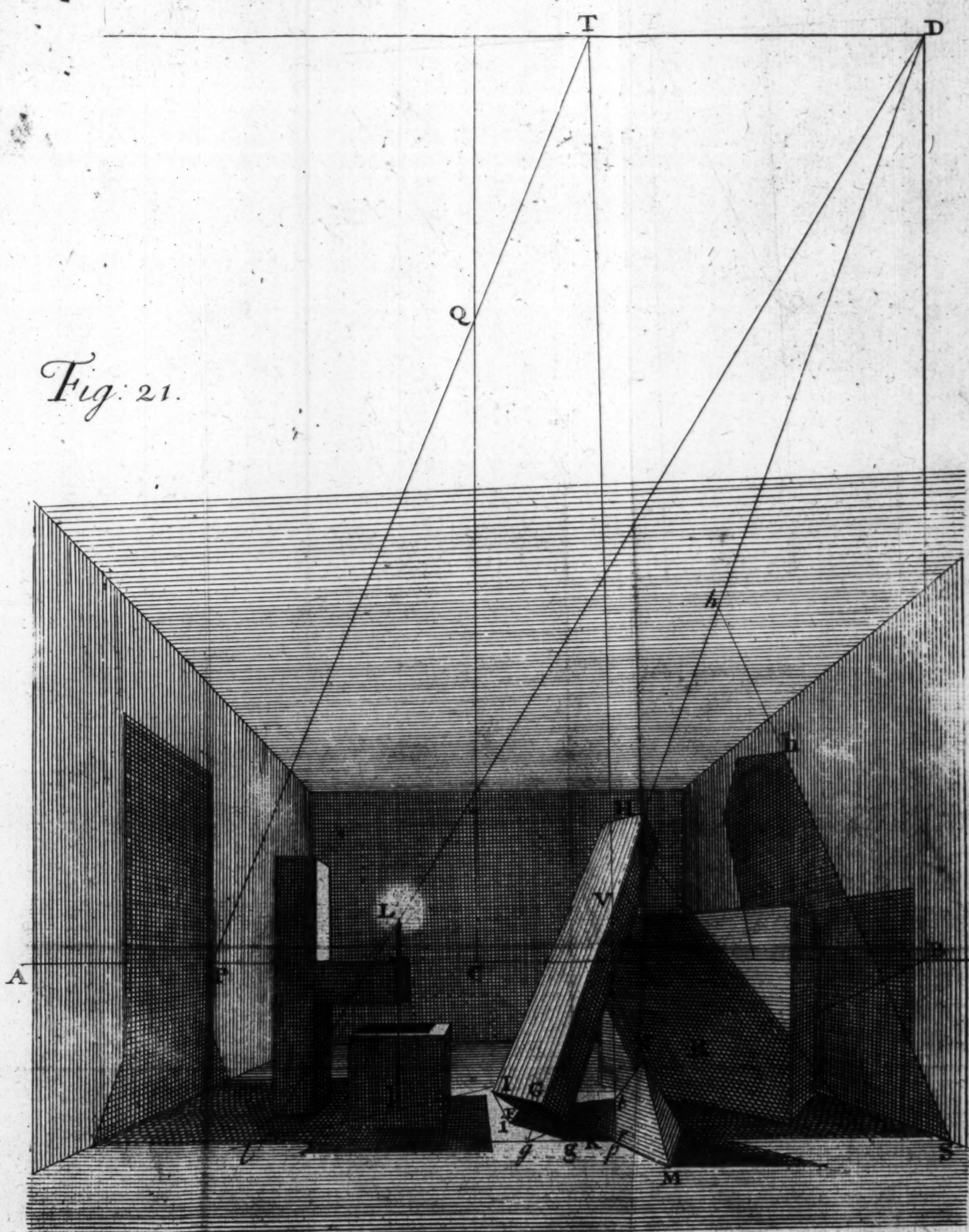


Fig. 23

